1. Consider the 2-D system of quasi-linear ODEs

\[
\frac{dx}{dt} = -2x + 2x^2 \\
\frac{dy}{dt} = -3x + y + 3x^2
\]

The goal of this problem is to find and classify all the equilibrium points of the system (i.e. classify each point as either stable or unstable and describe the behavior near the point as a saddle, center, spiral sink, spiral source, improper node, etc). NO COMPUTING DEVICES ALLOWED.

(a) 2 points. Find the Jacobian matrix of the system.

\[
J = \begin{pmatrix}
-2 + 4x & 0 \\
-3 + 6x & 1 
\end{pmatrix}
\]

(b) 2 points. Find the equilibrium points of the system.

\[
0 = -2x + 2x^2 \quad \Rightarrow \quad 2x(-1 + x) = 0 \\
0 = -3x + y + 3x^2 \quad \Rightarrow \quad 3x(-1 + x) = -y
\]

\[
x = 0 \text{ or } x = 1 \\
y = 0 \text{ or } y = 0
\]

(c) 6 points. Classify all the equilibrium points of the system.

\[
J(0,0) = \begin{pmatrix}
-2 & 0 \\
-3 & 1 
\end{pmatrix}
\]

\[D = \lambda_1\lambda_2 < 0 \Leftrightarrow \text{saddle}
\]

\[
\lambda = -2 \quad \text{and} \quad 1
\]

Since this is a diagonal matrix,

\[
(0, 0) \text{ behaves like a \textit{UNSTABLE saddle point}}
\]

\[
J(1,0) = \begin{pmatrix}
2 & 0 \\
3 & 1 
\end{pmatrix}
\]

\[\lambda = 2 \quad \text{and} \quad 1 > 0
\]

\[
(1, 0) \text{ behaves like an \textit{UNSTABLE SOURCE}}.
\]