Quiz **8** Ordinary Differential Equations

Name: ____________________ **Prof. Ron Buckmire**

ASSIGNED: Friday October 23

Time Begun: ____________

Time Ended: ____________ **DUE: Monday October 26**

Topic : Bifurcations in Systems of Differential Equations

The learning goal of this quiz is to provide you with an opportunity to combine your understanding of bifurcations with your understanding of linear systems of DEs.

Reality Check:

EXPECTED SCORE : ________**/10** ACTUAL SCORE : ________**/10**

Instructions:

0. Please look for a hint on this quiz posted to `http://sites.oxy.edu/ron/math/340/15/`
1. Once you open the quiz, you have **30 minutes** to complete it, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. **You must work alone**.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution. Use complete sentences wherever possible.
6. Relax and enjoy...
7. **This quiz is due at the beginning of class on Monday October 26**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the linear system of ordinary differential equations with a real-valued parameter a

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & a \\ 2 & 0 \end{bmatrix} \vec{x} \text{ where } \vec{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$$

Let's describe how the phase portrait changes as a varies from $-\infty$ to $+\infty$.

(a) *3 points.* Compute all the eigenvalues and eigenvectors in order to sketch the phase portrait when $a = -1$. Classify the stationary point at the origin (i.e. as unstable source, stable sink, or saddle point, etc) for this value of the parameter.

(b) *3 points.* Compute all the eigenvalues and eigenvectors in order to sketch the phase portrait when $a = 4$. Classify the stationary point at the origin for this value of the parameter.

(c) *4 points.* For what value of a does the system change its nature (i.e. bifurcate)? Call this value a_B and compute the eigenvalues and eigenvectors in order to sketch the phase portrait for $a = a_B$. Classify the stationary point at the origin for this value of the parameter.