Math 340 Fall 2015

Quiz **4** Ordinary Differential Equations

Name: _____ **Prof. Ron Buckmire**

ASSIGNED: Friday September 18

Time Begun: _____
Time Ended: _____ **DUE: Monday September 21**

Topic : Practice With Bifurcations

The learning goal of this quiz is to provide you with an opportunity to illustrate your understanding of bifurcations in first-order ordinary differential equations.

Reality Check:

EXPECTED SCORE : _____/**10** ACTUAL SCORE : _____/**10**

Instructions:

0. Please look for a hint on this quiz posted to `http://sites.oxy.edu/ron/math/340/15/`
1. Once you open the quiz, you have **30 minutes** to complete it, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. **You must work alone**.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution. Use complete sentences wherever possible.
6. Relax and enjoy...
7. **This quiz is due at the beginning of class on Monday September 21**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

Math 340 Fall 2015 **SHOW ALL YOUR WORK** Quiz **FOUR**

1. Consider the following one-parameter family of nonlinear, first-order, autonomous differential equations where α is a known real parameter value

$$\frac{dy}{dx} = y^2 - \alpha y + 1.$$

(a) *2 points.* Show that this DE has no equilibrium values when $|\alpha| < 2$.

(b) *2 points.* For what values of α will the DE have exactly one equilibrium value? Classify the equilibrium point (as node, source or sink) in this case and write down the constant solution.

(c) *4 points.* Show that when $|\alpha| > 2$ the DE has exactly one stable equilibrium value (sink) and one unstable equilibrium value (source). Show work that supports your classification of the equilibria, and sketch a phase line for a representative value of α.

(d) *2 points.* Use your answers from above to sketch the bifurcation diagram for the given DE. Clearly indicate on your sketch where sources, sinks and nodes occur. (HINT: think about what happens to the equilibrium values as $\alpha \to \pm\infty$!)