Quiz **2** ORDINARY DIFFERENTIAL EQUATIONS

Name: ______________________________ **Prof. Ron Buckmire**

Assigned: Friday September 4

Time Begun: ____________________
Time Ended: ____________________ **DUE: Wednesday September 9**

Topic : The Logistic Model of Population

The learning objective of this quiz is to provide the student with an opportunity to demonstrate your understanding of interpreting qualitative analysis of an ODE.

Reality Check:

EXPECTED SCORE : __________**/10** ACTUAL SCORE : __________**/10**

Instructions:

0. BEFORE you open the quiz, feel free to look for a hint at `faculty.oxy.edu/ron/math/340/15/`
1. Once you open the quiz, you have **30 minutes** to complete, please record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one. QUIZZES WITH UNSTAPLED SHEETS WILL NOT BE GRADED.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Relax and enjoy...
7. **This quiz is due at the beginning of class on Wednesday September 9**, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. Consider the initial value problem

$$\frac{dP}{dt} = kP(M - P), \quad P(0) = P_0$$

where $P(t)$ is the population in an environment which can only sustain M individuals, P_0 is the initial population size and k is related to the growth rate. (Note: k and M are known parameters of the Verhulst model.)

(a) *4 points.* Find the equilibrium solutions of the differential equation $P' = kP(M - P)$ and interpret the meaning of each solution for the real-world situation being described.

(b) *3 points.* Show that $\dfrac{d^2P}{dt^2} = 2k^2P(P - M)\left(P - \dfrac{M}{2}\right)$. (HINT: Use the chain rule!)

(c) *3 points.* Describe graphically (in words and pictures) what the future holds for a population for which it is determined that right now the current value of P_0 is $M/4$. In other words, use information about $P'(t)$ and $P''(t)$ to sketch a solution curve of $P' = kP(M - P)$ when $P(0) = M/4$.