

09.17.2014, Question 1: What is the equilibrium value of $\frac{dg}{dz} = -\frac{1}{2}g + 3e^z$?

- (a) This system is at equilibrium when $g = 6e^z$.
- (b) This system is at equilibrium when $z = \ln\left(\frac{g}{6}\right)$.
- (c) Both a and b are true.
- (d) This equation has no equilibrium.**

D

An equilibrium value of $g' = f(g, z)$ is a single value g^* of the dependent variable g which causes $f(g, z)$ to equal zero. It means that the solution $g(z) = g^*$ is an equilibrium solution of the ODE.

09.17.2014, Question 2: How many equilibria does the DE $y' = y^2 + a$ have?

A. Zero.

B. One.

C. Two.

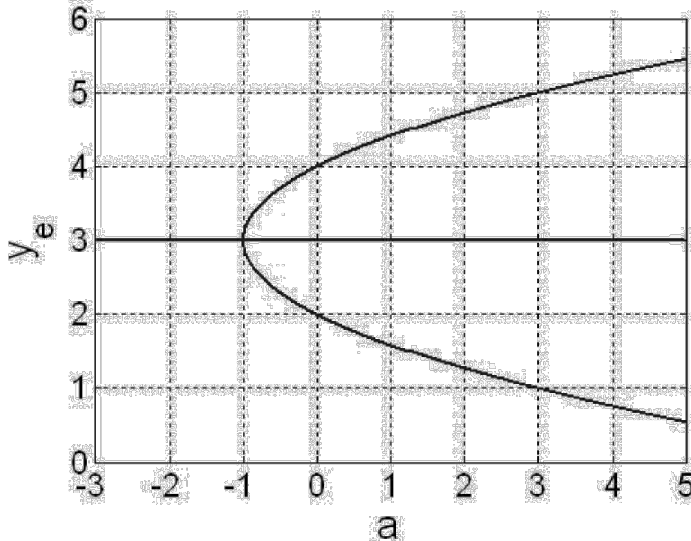
D. Three.

E. Not Enough Information Is Given.

E

You should recognize this ODE as having a saddle-nose bifurcation at $a=0$. In other words, depending on values of a there could be 2 ($a < 0$) or 1 ($a = 0$) or no equilibrium values ($a > 0$).

09.17.2014, Question 3: Consider the bifurcation diagram below. If the DE has equilibria at $y=1$, $y=3$, and $y=5$ what is the value of the bifurcation parameter a ?



A. $a=-1$

B. $a=0$

C. $a=1$

D. $a=3$

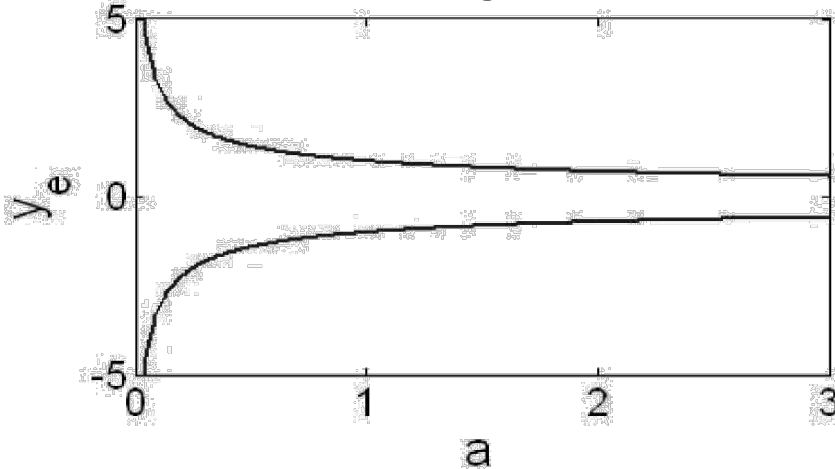
E. Not Enough Information
Is Given.

D

When the bifurcation parameter a equals 3 then there are three equilibria (At 1,3 and 5). The question is NOT asking you about the bifurcation value of the system, which in this case would be $a=0$.

09.17.2014, Question 4: Which of the following differential equations is represented by the bifurcation diagram below?

B



(a) $y' = y^2 + a$

(b) $y' = ay^2 - 1$

(c) $y' = ay$

(d) $y' = y^2 + ay + 2$

We can eliminate (a) and (c) since we know what bifurcation diagrams for those DEs would look like. Solving $0 = ay^2 - 1$ means $y^2 = 1/a$ or $y^* = -1/\sqrt{a}$ and $y^* = 1/\sqrt{a}$ which have the properties of the given curves in the figure (as $a \rightarrow 0$, y^* goes to infinity)