3. The equilibrium points occur at solutions of $dy/dt = y^2 - ay + 1 = 0$. From the quadratic formula, we have

$$y = \frac{a \pm \sqrt{a^2 - 4}}{2}.$$

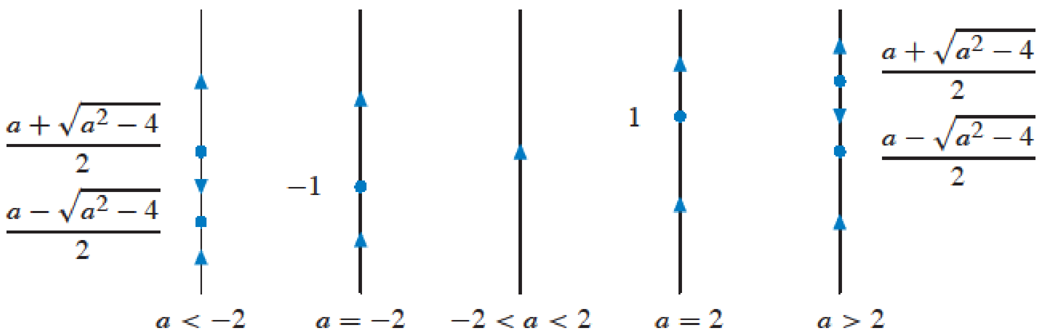
If $-2 < a < 2$, then $a^2 - 4 < 0$, and there are no equilibrium points. If $a > 2$ or $a < -2$, there are two equilibrium points. For $a = \pm 2$, there is one equilibrium point at $y = a/2$. The bifurcations occur at $a = \pm 2$.

To draw the phase lines, note that:

- For $-2 < a < 2$, $dy/dt = y^2 - ay + 1 > 0$, so the solutions are always increasing.
- For $a = 2$, $dy/dt = (y - 1)^2 \geq 0$, and $y = 1$ is a node.
- For $a = -2$, $dy/dt = (y + 1)^2 \geq 0$, and $y = -1$ is a node.
- For $a < -2$ or $a > 2$, let

$$y_1 = \frac{a - \sqrt{a^2 - 4}}{2} \quad \text{and} \quad y_2 = \frac{a + \sqrt{a^2 - 4}}{2}.$$

Then $dy/dt < 0$ if $y_1 < y < y_2$, and $dy/dt > 0$ if $y < y_1$ or $y > y_2$.

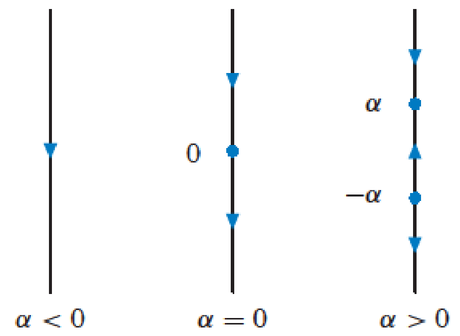


The five possible phase lines.

6. The equilibrium points occur at solutions of $dy/dt = \alpha - |y| = 0$. For $\alpha < 0$, there are no equilibrium points. For $\alpha = 0$, there is one equilibrium point, $y = 0$. For $\alpha > 0$, there are two equilibrium points, $y = \pm\alpha$. Therefore, $\alpha = 0$ is a bifurcation value.

To draw the phase lines, note that:

- If $\alpha < 0$, $dy/dt = \alpha - |y| < 0$, so the solutions are always decreasing.
- If $\alpha = 0$, $dy/dt < 0$ unless $y = 0$. Thus, $y = 0$ is a node.
- For $\alpha > 0$, $dy/dt > 0$ for $-\alpha < y < \alpha$, and $dy/dt < 0$ for $y < -\alpha$ and for $y > \alpha$.



10. Note that $0 < e^{-y^2} \leq 1$ for all y, and its maximum value occurs at $y = 0$. Therefore, for $\alpha < -1$, dy/dt is always negative, and the solutions are always decreasing.

If $\alpha = -1$, $dy/dt = 0$ if and only if $y = 0$. For $y \neq 0$, $dy/dt < 0$, and the equilibrium point at $y = 0$ is a node.

If $-1 < \alpha < 0$, then there are two equilibrium points which we compute by solving

$$e^{-y^2} + \alpha = 0.$$

We get $-y^2 = \ln(-\alpha)$. Consequently, $y = \pm\sqrt{\ln(-1/\alpha)}$. As $\alpha \to 0$ from below, $\ln(-1/\alpha) \to \infty$, and the two equilibria tend to $\pm\infty$.

If $\alpha \geq 0$, dy/dt is always positive, and the solutions are always increasing.

13. **(a)** Each phase line has an equilibrium point at $y = 0$. This corresponds to equations (i), (iii), and (vi). Since $y = 0$ is the only equilibrium point for $A < 0$, this only corresponds to equation (iii).

(b) The phase line corresponding to $A = 0$ is the only phase line with $y = 0$ as an equilibrium point, which corresponds to equations (ii), (iv), and (v). For the phase lines corresponding to $A < 0$, there are no equilibrium points. Only equations (iv) and (v) satisfy this property. For the phase lines corresponding to $A > 0$, note that $dy/dt < 0$ for $-\sqrt{A} < y < \sqrt{A}$. Consequently, the bifurcation diagram corresponds to equation (v).

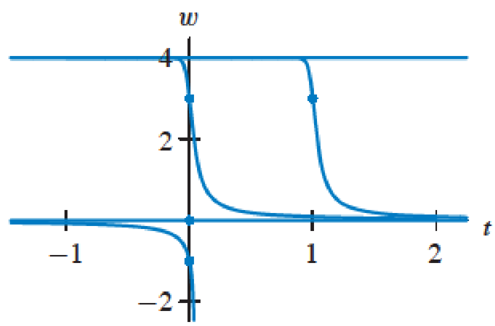
(c) The phase line corresponding to $A = 0$ is the only phase line with $y = 0$ as an equilibrium point, which corresponds to equations (ii), (iv), and (v). For the phase lines corresponding to $A < 0$, there are no equilibrium points. Only equations (iv) and (v) satisfy this property. For the phase lines corresponding to $A > 0$, note that $dy/dt > 0$ for $-\sqrt{A} < y < \sqrt{A}$. Consequently, the bifurcation diagram corresponds to equation (iv).

(d) Each phase line has an equilibrium point at $y = 0$. This corresponds to equations (i), (iii), and (vi). The phase lines corresponding to $A > 0$ only have two nonnegative equilibrium points. Consequently, the bifurcation diagram corresponds to equation (i).

18. **(a)** For all $C \geq 0$, the equation has a source at $P = C/k$, and this is the only equilibrium point. Hence all of the phase lines are qualitatively the same, and there are no bifurcation values for C.

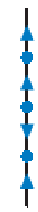
(b) If $P(0) > C/k$, the corresponding solution $P(t) \to \infty$ at an exponential rate as $t \to \infty$, and if $P(0) < C/k$, $P(t) \to -\infty$, passing through "extinction" ($P = 0$) after a finite time.

20.



30. The function $f(y)$ has two zeros, one positive and one negative. We denote them as y_1 and y_2, where $y_1 < y_2$. So the differential equation $dy/dt = f(y)$ has two equilibrium solutions, one for each zero. Also, $f(y) > 0$ if $y_1 < y < y_2$ and $f(y) < 0$ if $y < y_1$ or if $y > y_2$. Hence y_1 is a source and y_2 is a sink.

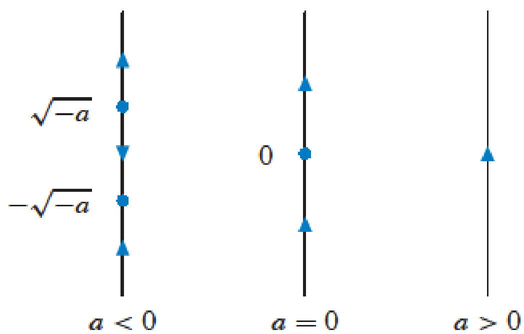
31. The function $f(y)$ has three zeros. We denote them as y_1, y_2, and y_3, where $y_1 < 0 < y_2 < y_3$. So the differential equation $dy/dt = f(y)$ has three equilibrium solutions, one for each zero. Also, $f(y) > 0$ if $y < y_1$, $f(y) < 0$ if $y_1 < y < y_2$, and $f(y) > 0$ if $y_2 < y < y_3$ or if $y > y_3$. Hence y_1 is a sink, y_2 is a source, and y_3 is a node.



41. The equilibrium points occur at solutions of $dy/dt = y^2 + a = 0$. For $a > 0$, there are no equilibrium points. For $a = 0$, there is one equilibrium point, $y = 0$. For $a < 0$, there are two equilibrium points, $y = \pm\sqrt{-a}$.

To draw the phase lines, note that:

- If $a > 0$, $dy/dt = y^2 + a > 0$, so the solutions are always increasing.
- If $a = 0$, $dy/dt > 0$ unless $y = 0$. Thus, $y = 0$ is a node.
- For $a < 0$, $dy/dt < 0$ for $-\sqrt{-a} < y < \sqrt{-a}$, and $dy/dt > 0$ for $y < -\sqrt{-a}$ and for $y > \sqrt{-a}$.



(a) The phase lines for $a < 0$ are qualitatively the same, and the phase lines for $a > 0$ are qualitatively the same.

(b) The phase line undergoes a qualitative change at $a = 0$.