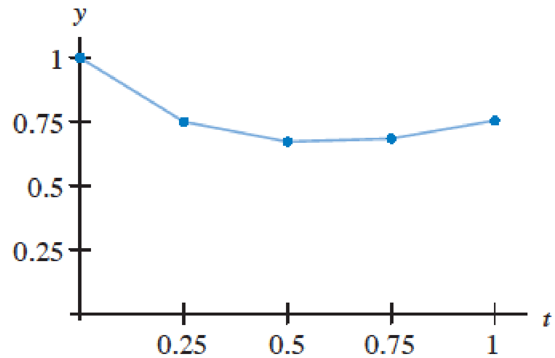
2.

Table 1.2
Results of Euler's method (y_k rounded to two decimal places)

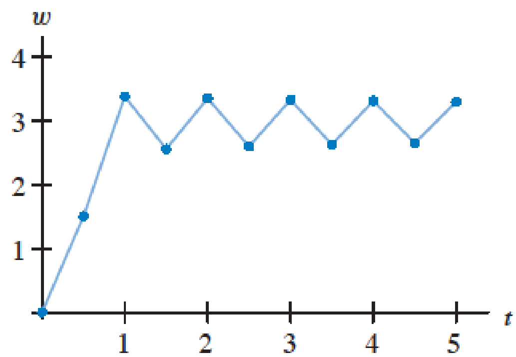
k	t_k	y_k	m_k
0	0	1	-1
1	0.25	0.75	-0.3125
2	0.5	0.67	0.0485
3	0.75	0.68	0.282
4	1.0	0.75	



6.

Table 1.6
Results of Euler's method (shown rounded to two decimal places)

k	t_k	w_k	m_k
0	0	0	3
1	0.5	1.5	3.75
2	1.0	3.38	−1.64
3	1.5	2.55	1.58
4	2.0	3.35	−1.50
5	2.5	2.59	1.46
6	3.0	3.32	−1.40
7	3.5	2.62	1.36
8	4.0	3.31	−1.31
9	4.5	2.65	1.28
10	5.0	3.29	



11. As the solution approaches the equilibrium solution corresponding to $w = 3$, its slope decreases. We do not expect the solution to "jump over" an equilibrium solution (see the Existence and Uniqueness Theorem in Section 1.5).

15.

Table 1.13
Results of Euler's method with $\Delta t = 1.0$ (shown to two decimal places)

k	t_k	y_k	m_k
0	0	1	1
1	1	2	1.41
2	2	3.41	1.85
3	3	5.26	2.29
4	4	7.56	

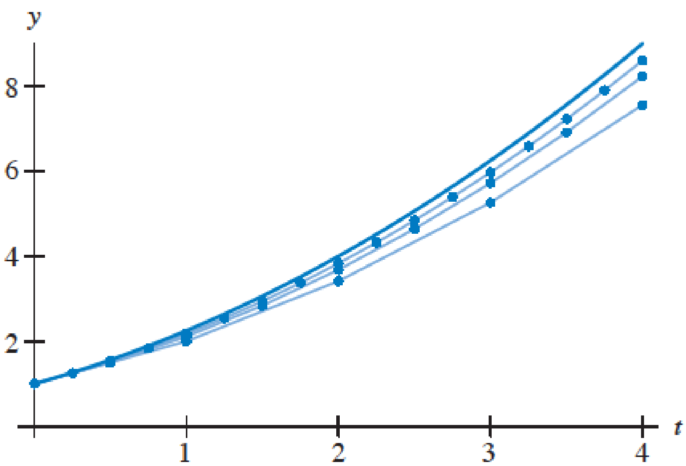
Table 1.14
Results of Euler's method with $\Delta t = 0.5$ (shown to two decimal places)

k	t_k	y_k	m_k	k	t_k	y_k	m_k
0	0	1	1	5	2.5	4.64	2.15
1	0.5	1.5	1.22	6	3.0	5.72	2.39
2	1.0	2.11	1.45	7	3.5	6.91	2.63
3	1.5	2.84	1.68	8	4.0	8.23	
4	2.0	3.68	1.92				

Table 1.15
Results of Euler's method with $\Delta t = 0.25$ (shown to two decimal places)

k	t_k	y_k	m_k	k	t_k	y_k	m_k
0	0	1	1	9	2.25	4.32	2.08
1	0.25	1.25	1.12	10	2.50	4.84	2.20
2	0.50	1.53	1.24	11	2.75	5.39	2.32
3	0.75	1.84	1.36	12	3.0	5.97	2.44
4	1.0	2.18	1.48	13	3.25	6.58	2.56
5	1.25	2.55	1.60	14	3.50	7.23	2.69
6	1.50	2.94	1.72	15	3.75	7.90	2.81
7	1.75	3.37	1.84	16	4.0	8.60	
8	2.0	3.83	1.96				

The slopes in the slope field are positive and increasing. Hence, the graphs of all solutions are concave up. Since Euler's method uses line segments to approximate the graph of the actual solution, the approximate solutions will always be less than the actual solution. This error decreases as the step size decreases.



2. Since $y(0) = 1$ is between the equilibrium solutions $y_2(t) = 0$ and $y_3(t) = 2$, we must have $0 < y(t) < 2$ for all t because the Uniqueness Theorem implies that graphs of solutions cannot cross (or even touch in this case).

3. Because $y_2(0) < y(0) < y_1(0)$, we know that

$$-t^2 = y_2(t) < y(t) < y_1(t) = t + 2$$

for all t. This restricts how large positive or negative $y(t)$ can be for a given value of t (that is, between $-t^2$ and $t + 2$). As $t \to -\infty$, $y(t) \to -\infty$ between $-t^2$ and $t + 2$ ($y(t) \to -\infty$ as $t \to -\infty$ at least linearly, but no faster than quadratically).

12. **(a)** Note that

$$\frac{dy_1}{dt} = \frac{d}{dt}\left(\frac{1}{t-1}\right) = -\frac{1}{(t-1)^2} = -(y_1(t))^2$$

and

$$\frac{dy_2}{dt} = \frac{d}{dt}\left(\frac{1}{t-2}\right) = -\frac{1}{(t-2)^2} = -(y_2(t))^2,$$

so both $y_1(t)$ and $y_2(t)$ are solutions.

(b) Note that $y_1(0) = -1$ and $y_2(0) = -1/2$. If $y(t)$ is another solution whose initial condition satisfies $-1 < y(0) < -1/2$, then $y_1(t) < y(t) < y_2(t)$ for all t by the Uniqueness Theorem. Also, since $dy/dt < 0$, $y(t)$ is decreasing for all t in its domain. Therefore, $y(t) \to 0$ as $t \to -\infty$, and the graph of $y(t)$ has a vertical asymptote between $t = 1$ and $t = 2$.

14. **(a)** The equation is separable, so we obtain

$$\int (y+1)\,dy = \int \frac{dt}{t-2}.$$

Solving for y with help from the quadratic formula yields the general solution

$$y(t) = -1 \pm \sqrt{1+\ln(c(t-2)^2)}$$

where c is a constant. Substituting the initial condition $y(0)=0$ and solving for c, we have

$$0 = -1 \pm \sqrt{1+\ln(4c)},$$

and thus $c = 1/4$. The desired solution is therefore

$$y(t) = -1 + \sqrt{1+\ln((1-t/2)^2)}$$

(b) The solution is defined only when $1+\ln((1-t/2)^2) \geq 0$, that is, when $|t-2| \geq 2/\sqrt{e}$. Therefore, the domain of the solution is

$$t \leq 2(1-1/\sqrt{e}).$$

(c) As $t \to 2(1-1/\sqrt{e})$, then $1+\ln((1-t/2)^2) \to 0$. Thus

$$\lim_{t\to 2(1-1/\sqrt{e})} y(t) = -1.$$

Note that the differential equation is not defined at $y=-1$. Also, note that

$$\lim_{t\to -\infty} y(t) = \infty.$$

15. **(a)** The equation is separable. We separate, integrate

$$\int (y+2)^2\, dy = \int dt,$$

and solve for y to obtain the general solution

$$y(t) = (3t + c)^{1/3} - 2,$$

where c is any constant. To obtain the desired solution, we use the initial condition $y(0) = 1$ and solve

$$1 = (3 \cdot 0 + c)^{1/3} - 2$$

for c to obtain $c = 27$. So the solution to the given initial-value problem is

$$y(t) = (3t + 27)^{1/3} - 2.$$

(b) This function is defined for all t. However, $y(-9) = -2$, and the differential equation is not defined at $y = -2$. Strictly speaking, the solution exists only for $t > -9$.

(c) As $t \to \infty$, $y(t) \to \infty$. As $t \to -9^+$, $y(t) \to -2$.

2. The equilibrium points of $dy/dt = f(y)$ are the numbers y where $f(y) = 0$. For $f(y) = y^2 - 4y - 12 = (y-6)(y+2)$, the equilibrium points are $y = -2$ and $y = 6$. Since $f(y)$ is positive for $y < -2$, negative for $-2 < y < 6$, and positive for $y > 6$, the equilibrium point $y = -2$ is a sink and the equilibrium point $y = 6$ is a source.