1. **(a)** Let's check Bob's solution first. Since $dy/dt = 1$ and

$$\frac{y(t)+1}{t+1} = \frac{t+1}{t+1} = 1,$$

Bob's answer is correct.

Now let's check Glen's solution. Since $dy/dt = 2$ and

$$\frac{y(t)+1}{t+1} = \frac{2t+2}{t+1} = 2,$$

Glen's solution is also correct.

Finally let's check Paul's solution. We have $dy/dt = 2t$ on one hand and

$$\frac{y(t)+1}{t+1} = \frac{t^2-1}{t+1} = t-1$$

on the other. Paul is wrong.

(b) At first glance, they should have seen the equilibrium solution $y(t) = -1$ for all t because $dy/dt = 0$ for any constant function and $y = -1$ implies that

$$\frac{y+1}{t+1} = 0$$

independent of t.

Strictly speaking the differential equation is not defined for $t = -1$, and hence the solutions are not defined for $t = -1$.

2. We note that $dy/dt = 2e^{2t}$ for $y(t) = e^{2t}$. If $y(t) = e^{2t}$ is a solution to the differential equation, then we must have

$$\begin{aligned} 2e^{2t} &= 2y(t) - t + g(y(t)) \\ &= 2e^{2t} - t + g(e^{2t}). \end{aligned}$$

Hence, we need

$$g(e^{2t}) = t.$$

This equation is satisfied if we let $g(y) = (\ln y)/2$. In other words, $y(t) = e^{2t}$ is a solution of the differential equation

$$\frac{dy}{dt} = 2y - t + \frac{\ln y}{2}.$$

3. In order to find one such $f(t, y)$, we compute the derivative of $y(t)$. We obtain

$$\frac{dy}{dt} = \frac{de^{t^3}}{dt} = 3t^2e^{t^3}.$$

Now we replace e^{t^3} in the last expression by y and get the differential equation

$$\frac{dy}{dt} = 3t^2y.$$

6. Separating variables and integrating, we obtain

$$\int \frac{1}{y}\,dy = \int t^4\,dt$$

$$\ln|y| = \frac{t^5}{5} + c$$

$$|y| = c_1 e^{t^5/5},$$

where $c_1 = e^c$. As in Exercise 22, we can eliminate the absolute values by replacing the positive constant c_1 with $k = \pm c_1$. Hence, the general solution is

$$y(t) = k e^{t^5/5},$$

where k is any real number. Note that $k = 0$ gives the equilibrium solution.

21. The function $y(t) = 0$ for all t is an equilibrium solution.

Suppose $y \neq 0$ and separate variables. We get

$$\int y + \frac{1}{y}\,dy = \int e^t\,dt$$

$$\frac{y^2}{2} + \ln|y| = e^t + c,$$

where c is any real constant. We cannot solve this equation for y, so we leave the expression for y in this implicit form. Note that the equilibrium solution $y = 0$ cannot be obtained from this implicit equation.

27. Separating variables and integrating, we obtain

$$\int \frac{dy}{y^2} = -\int dt$$

$$-\frac{1}{y} = -t + c.$$

So we get

$$y = \frac{1}{t - c}.$$

Now we need to find the constant c so that $y(0) = 1/2$. To do this we solve

$$\frac{1}{2} = \frac{1}{0 - c}$$

and get $c = -2$. The solution of the initial-value problem is

$$y(t) = \frac{1}{t + 2}.$$

32. First we find the general solution by writing the differential equation as

$$\frac{dy}{dt} = (t+2)y^2,$$

separating variables, and integrating. We have

$$\int \frac{1}{y^2}\,dy = \int (t+2)\,dt$$

$$-\frac{1}{y} = \frac{t^2}{2} + 2t + c$$

$$= \frac{t^2 + 4t + c_1}{2},$$

where $c_1 = 2c$. Inverting and multiplying by -1 produces

$$y(t) = \frac{-2}{t^2 + 4t + c_1}.$$

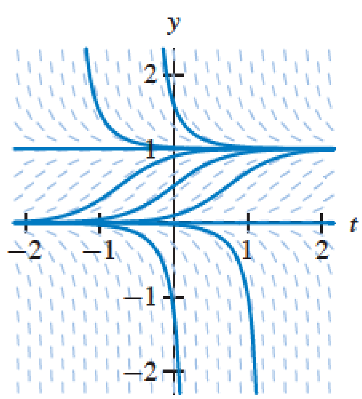
Setting

$$1 = y(0) = \frac{-2}{c_1}$$

and solving for c_1, we obtain $c_1 = -2$. So

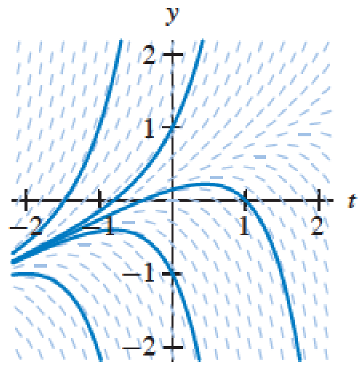
$$y(t) = \frac{-2}{t^2 + 4t - 2}.$$

7. **(a)**



(b) The solution with $y(0) = 1/2$ approaches the equilibrium value $y = 1$ from below as t increases. It decreases toward $y = 0$ as t decreases.

8. **(a)**



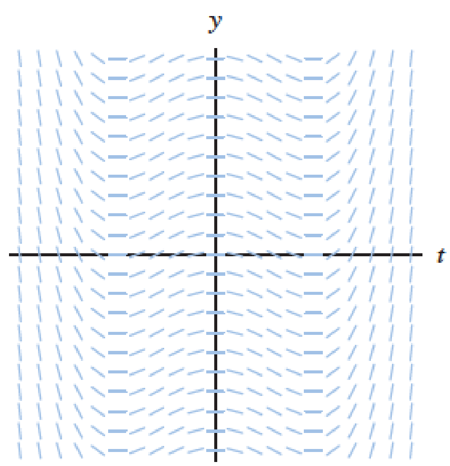
(b) The solution $y(t)$ with $y(0) = 1/2$ increases with $y(t) \to \infty$ as t increases. As t decreases, $y(t) \to -\infty$.

12. **(a)** Since $y(t) = 2$ for all t is a solution and $dy/dt = 0$ for all t, $f(t, y(t)) = f(t, 2) = 0$ for all t.

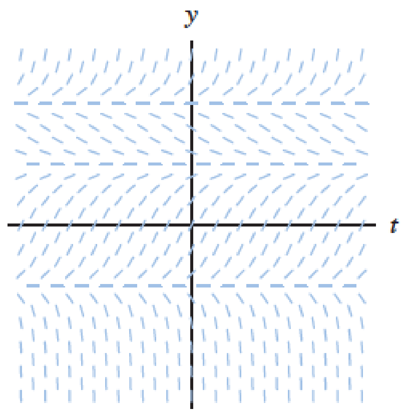
(b) Therefore, the slope marks all have zero slope along the horizontal line $y = 2$.

(c) If the graphs of solutions cannot cross in the ty-plane, then the graph of a solution must stay on the same side of the line $y = 2$ as it is at time $t = 0$. In Section 1.5, we discuss conditions that guarantee that graphs of solutions do not cross.

13. The slope field in the ty-plane is constant along vertical lines.



14. Because f depends only on y (the equation is autonomous), the slope field is constant along horizontal lines in the ty-plane. The roots of f correspond to equilibrium solutions. If $f(y) > 0$, the corresponding lines in the slope field have positive slope. If $f(y) < 0$, the corresponding lines in the slope field have negative slope.



16. **(a)** This slope field is constant along horizontal lines, so it corresponds to an autonomous equation. The autonomous equations are (i), (ii), and (iii). This field does not correspond to equation (ii) because it has the equilibrium solution $y = -1$. The slopes are negative for $y < -1$. Consequently, this field corresponds to equation (iii).

(b) Note that the slopes are constant along vertical lines—lines along which t is constant, so the right-hand side of the corresponding equation depends only on t. The only choices are equations (iv) and (viii). Since the slopes are negative for $-\sqrt{2} < t < \sqrt{2}$, this slope field corresponds to equation (viii).

(c) This slope field depends both on y and on t, so it can only correspond to equations (v), (vi), or (vii). Since this field has the equilibrium solution $y = 0$, this slope field corresponds to equation (v).

(d) This slope field also depends on both y and on t, so it can only correspond to equations (v), (vi), or (vii). This field does not correspond to equation (v) because $y = 0$ is not an equilibrium solution. Since the slopes are nonnegative for $y > -1$, this slope field corresponds to equation (vi).