

2. Note that $dy/dt = 0$ for all t only if $y^2 - 2 = 0$. Therefore, the only equilibrium solutions are $y(t) = -\sqrt{2}$ for all t and $y(t) = +\sqrt{2}$ for all t .

3. (a) The equilibrium solutions correspond to the values of P for which $dP/dt = 0$ for all t . For this equation, $dP/dt = 0$ for all t if $P = 0$ or $P = 230$.

(b) The population is increasing if $dP/dt > 0$. That is, $P(1 - P/230) > 0$. Hence, $0 < P < 230$.

(c) The population is decreasing if $dP/dt < 0$. That is, $P(1 - P/230) < 0$. Hence, $P > 230$ or $P < 0$. Since this is a population model, $P < 0$ might be considered “nonphysical.”

13. The rate of learning is dL/dt . Thus, we want to know the values of L between 0 and 1 for which dL/dt is a maximum. As $k > 0$ and $dL/dt = k(1 - L)$, dL/dt attains its maximum value at $L = 0$.

14. (a) Let $L_1(t)$ be the solution of the model with $L_1(0) = 1/2$ (the student who starts out knowing one-half of the list) and $L_2(t)$ be the solution of the model with $L_2(0) = 0$ (the student who starts out knowing none of the list). At time $t = 0$,

$$\frac{dL_1}{dt} = 2(1 - L_1(0)) = 2\left(1 - \frac{1}{2}\right) = 1,$$

and

$$\frac{dL_2}{dt} = 2(1 - L_2(0)) = 2.$$

Hence, the student who starts out knowing none of the list learns faster at time $t = 0$.

(b) The solution $L_2(t)$ with $L_2(0) = 0$ will learn one-half the list in some amount of time $t_* > 0$. For $t > t_*$, $L_2(t)$ will increase at exactly the same rate that $L_1(t)$ increases for $t > 0$. In other words, $L_2(t)$ increases at the same rate as $L_1(t)$ at t_* time units later. Hence, $L_2(t)$ will never catch up to $L_1(t)$ (although they both approach 1 as t increases). In other words, after a very long time $L_2(t) \approx L_1(t)$, but $L_2(t) < L_1(t)$.