Complex Analysis

Math 312 MWF 10:30am - 11:25am
 Fowler 112

Class 30 **(Wednesday April 8)**

SUMMARY Using Laurent Series to Compute Residues
CURRENT READING Brown & Curchill pages 154–158
NEXT READING Brown & Curchill pages 210–219

Laurent Series

$$f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n + \sum_{n=1}^{\infty} \frac{b_n}{(z-z_0)^n}, \qquad R_1 < |z-z_0| < R_2$$

This formula for a Laurent series is also sometimes written as

$$f(z) = \sum_{n=-\infty}^{\infty} c_n (z-z_0)^n \qquad \text{where } c_n = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-z_0)^{n+1}} dz, \quad n = 0, \pm 1, \pm 2, \ldots$$

In practice, one usually computes a Laurent series by comparing the function you have to one of the "famous functions" whose Maclaurin series you have memorized.

GROUPWORK

1. Write down the Laurent series for $z^2 \sin(1/z^2)$ in the domain $0 < |z| < \infty$

2. What is the value of $\text{Res}(z^2 \sin(1/z^2), 0)$?

3. Evaluate $\oint_{|z|=1} z^2 \sin(1/z^2) dz$

4. Write down two different Laurent series for $f(z) = \dfrac{1}{z(z^2+1)}$ and specify the domain in which your series are valid.

5. Evaluate $\displaystyle\oint_{|z|=2} \frac{1}{z(z^2+1)} dz$ using whichever method you prefer.