# Complex Analysis

Math 312 Spring 1998 — MWF 10:30am - 11:25am
**Buckmire** — Fowler 112

*Class 22* **(Monday March 9)**

**SUMMARY** Path Independence and the Cauchy-Goursat Theorem
**CURRENT READING** Brown & Curchill pages 104-111
**NEXT READING** Brown & Curchill pages 116-123

We have been practicing our complex integration skills. Today we will learn how to use **antiderivatives** and the **Cauchy-Goursat Theorem** to evaluate contour integrals more efficiently (simply!)
Recall that the value of $\int_C 2\overline{z}^2\, dz$ **did** depend on the contour when we evaluated it in the previous class.
However the value of $\int_C 2z^2\, dz$ **did not** depend on the contour chosen and in each case the integral is equal to $\frac{-32}{3}$
How is $f(z) = 2z^2$ a different function than $g(z) = 2\overline{z}^2$?
$2z^2$ is a ____________________ while $2\overline{z}^2$ is a ____________________.

**Independence of Path**
**THEOREM:** Suppose that the function $f(z)$ is continuous in a domain (open connected set) $D$ and has an antiderivative $F(z)$ throughout $D$, i.e. $dF/dz = f(z)$ at each point in $D$. Then for *every* contour , lying in $D$ connecting $z_1$ to $z_2$ we have the result

$$\int_\Gamma f(z)\, dz = F(z_2) - F(z_1)$$

In otherwords, the value of the integral is **independent of the path** chosen to link $z_1$ and $z_2$
So, getting back to the difference between $f(z) = 2z^2$ and $g(z) = 2\overline{z}^2$ we know that $2z^2$ has the property that it is continuous and is equal to the derivative of $2z^3/3$ on the open set $z \in \mathbf{C}$. $g(z)$ has no similar antiderivative. (This is not surprising, since $g(z)$ doesn't have a derivative either.)

**Example**
$\int_C 2z^2\, dz =$ where $C$ is a contour from 2 to $-2$

Note that according to the above theorem the function $F(z)$ will be **analytic** and **continuous** on the domain $D$ (since it has a derivative at every point of the open set $D$).
A corollary of the above theorem is the famous

**Cauchy-Goursat Theorem:** If $f(z)$ is analytic at all points interior to and on any simple closed contour , , then

$$\oint_{\Gamma} f(z)dz = 0.$$

(Is this result surprising?)

GROUPWORK

1. $\int_C \sin(iz) + 2e^z\, dz,$ where $C$is a contour joining 0 to $\pi i$

2. $\oint_C \frac{dz}{z^2-4}\, dz,$ where $C : |z| = 1$

3. $\int_{-i}^{i} \frac{dz}{z^2}$ and $\oint_{C_r} \frac{dz}{z^2}$ where $C_r : |z| = 1$