Complex Analysis

Math 312 Spring 1998 — MWF 10:30am - 11:25am
Buckmire — Fowler 112

Class 21 **(Friday March 6)**

SUMMARY Examples of Contour Integration
CURRENT READING Brown & Curchill, pages 104-122
NEXT READING Brown & Curchill pages 104-122

Parametricization of a line segment
Can you come up with a formula for a parametrization $z(t)$ of a directed line segment from point z_1 to z_2 where t starts at t_1 and ends at t_2 so that $z(t_1) = z_1$ and $z(t_2) = z_2$? Write it down below:

Exercise
Consider $\int_C 2z^2\,dz$ where C is the directed line segment from $z = 2$ to $z = -2$ (Sketch the contour and evaluate the integral.)

GROUPWORK
Again evaluate $\int_C 2z^2\,dz$ where C is the circular arc going from $z = 2$ to $z = -2$. (Sketch the contour and evaluate the integral.)

SUPPLEMENTARY EXERCISE

If you want more practice evaluating contours you should try to evaluate $\int_C 2z^2\,dz$ where C is the directed line segment going from $z = 2$ to $z = -2$ via the point $z = 2i$. (Sketch the contour and evaluate the integral.)

Question

What's the difference between the integral $\int_{-2}^{2} 2\overline{z}^2\,dz$ and $\int_{-2}^{2} 2z^2\,dz$?
Does the value of a contour integral depend on the path taken?

Does path dependence of a contour integral depend on the function involved? What property of the function is involved?

Proprties of Contour Integrals

$$\int_C f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz$$

$$\int_{-C} f(z)dz = -\int_C f(z)dz$$

$$\int_C z_0 f(z)dz = z_0 \int_C f(z)dz, \qquad z_0 \in \mathbf{C}$$

$$\int_C f(z) + g(z)dz = \int_C f(z)dz + \int_C g(z)dz$$

$$\left|\int_C f(z)dz\right| \leq ML$$

where L is the length of the contour and $\int_a^b |z'(t)|dt \leq L$
and M is an upper bound on $f(z)$, $|f(z)| \leq M$

Exercise

If is the arc of the circle $|z| = 2$ traversed in the counter-clockwise direction, then we want to show that

$$\left|\int \frac{e^z}{z^2+1}dz\right| \leq \frac{4\pi e^2}{3}$$