# Complex Analysis

Math 312 Spring 1998 — MWF 10:30am - 11:25am
**Buckmire** — Fowler 112

*Class 20* **(Wednesday March 4)**

**SUMMARY** Introduction to Contour Integration
**CURRENT READING** Brown & Curchill, pages 95-104
**NEXT READING** Brown & Curchill pages 104-122

**Exercise**
First, let's recall how to integrate complex functions of a **real** variable. Compute the following:

$$(a)\int_1^2 \frac{-i}{t^2} + (t+2i)^3\,dt \qquad\qquad (b)\int_0^\infty e^{-z^2 t}\,dt$$

**Contour Integration**
Integration of a complex function of a **complex** variable is performed on a set of connected points from, say, $z_1$ to $z_2$. It is a **contour integral**. Given a contour $C$ defined as $z(t)$ for $a \le t \le b$ where $z_1 = z(a)$ and $z_2 = z(b)$, an integral of a complex function of a complex variable $f(z)$ is written as

$$\int_C f(z)dz \qquad \text{or} \qquad \int_{z_1}^{z_2} f(z)\;dz$$

Let $f(z)$ be piecewise continuous on $z(t)$. If $C$ is a **contour** then $z'(t)$ is piecewise continuous on $a \le t \le b$ and we can redefine the integral of $f(z)$ along $C$ as:

$$\int_C f(z)\;dz = \int_a^b f[z(t)]z'(t)\;dt$$

**Examples**
Compute $\displaystyle\int_C \text{Re}\; z\; dz$ where $C$ is a directed line segment from $z = 0$ to $z = 1 + 2i$
ALGORITHM: steps to be taken to complete the process
**1**: write down a parametricization for $C$, $z(t)$
**2**: Convert the integral into an integral in (real) $t$ variables
**3**: Integrate!

## GROUPWORK

Compute $\int_C 2\overline{z}^2\ dz$ where $C$ is a directed l ine segment from $z = 2$ to $z = -2$. (Sketch the contour and evaluate the integral.)

Also $\int_C 2\overline{z}^2\ dz$, this time using $C$ being a circular arc from $z = 2$ to $z = -2$. (Sketch the contour and then evaluate the integral.)

**Exercise**

Show that

$$\int_{C_r} (z - z_0)^n\ dz = \begin{cases} 2\pi i & n = -1 \\ \\ 0 & n \neq -1 \end{cases}$$

where $n$ is any integer and $C_r$ is a circle of radius $r$ around $z_0$ (what is the equation of such a shape?) traversed **once** in the counter-clockwise direction