Complex Analysis

Math 312 Spring 1998 — MWF 10:30am - 11:25am
Buckmire — Fowler 112

Class #6 (Monday January 26)

SUMMARY Functions of a Complex Variable

CURRENT READING Brown & Churchill, pages 26-32

NEXT READING Brown & Churchill, pages 33-38

Functions of a Complex Variable

Given a set S of complex numbers, a function f is a rule which assigns to each $z \in S$ a complex number w. The set S is knows as the **domain of definition** of f. (NOTE: The "domain of definition" is not necessarily a 'bf domain in the formal mathematical sense of the word.) The set of all $w \in \mathcal{Z}$ given by $w = f(S)$ is called the **image of S under f** or the **range of f.**
The value $w = f(z)$ can be written as $u + iv$, in other words:

$$w = f(z) = f(x + iy) = u(x, y) + iv(x, y)$$

where $u(x, y)$ and $v(x, y)$ are real functions of two real variables.
<u>**Example**</u>
Write $f(z) = z^2 - z + 2i$ in the form $w = u(x, y) + iv(x, y)$

In addition, given a function $w(x, y)$ you can write it in terms of z, $\overline{z}$ and constants.
<u>**Example**</u>
Write $w(x, y) = x^2 + iy^2$ in terms of z and $\overline{z}$

Mapping

As usual, operations using complex variables have geometric significance.
First, let's get more practice evaluating functions of a complex variable:
Using $f(z) = z^3$, compute

(a) $f(2) =$

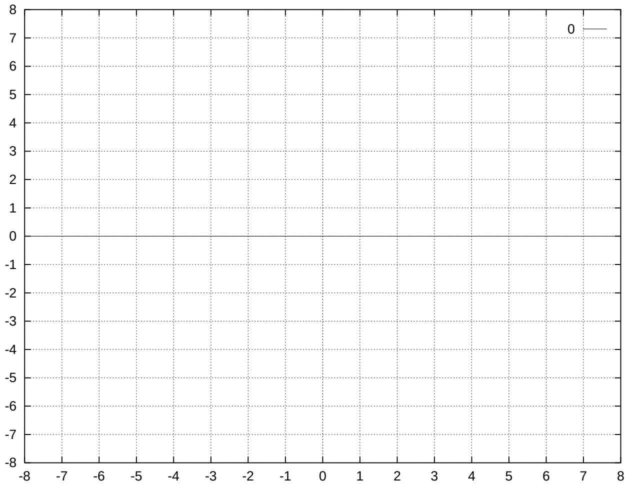
(b) $f(\sqrt{2} + i\sqrt{2}) =$

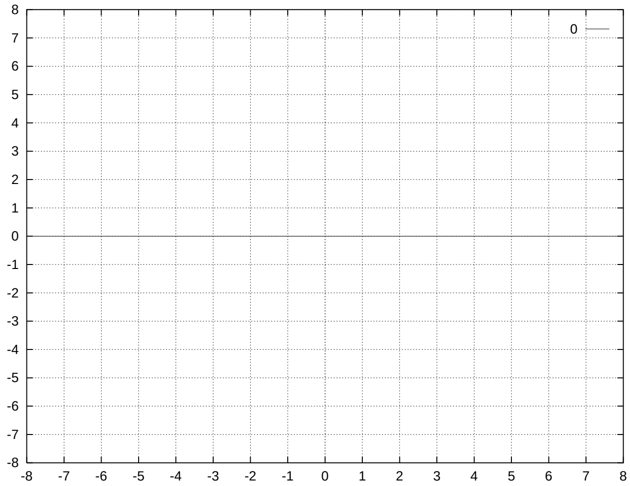
(c) $f(2i) =$

Example

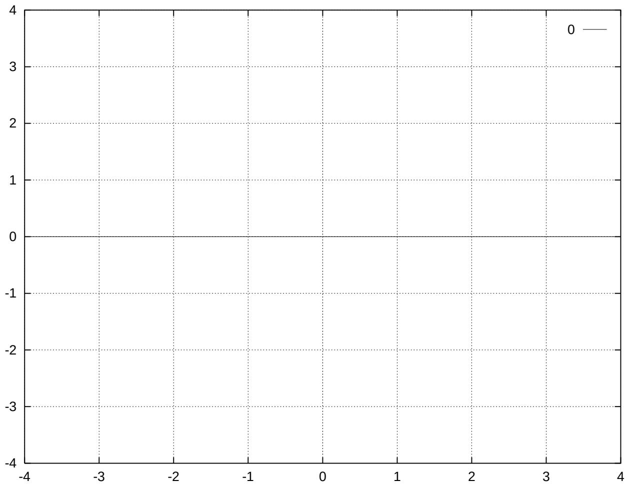
If you consider $w = f(z)$ a mapping from the z-plane to the w-plane, draw a sketch of what the "quarter-disc" of radius 2 in the first quadrant of the z-plane maps to in the w-plane under the mapping $f(z) = z^3$.
In the language of mathematics, we say that what you draw in the w-plane is **the image** of the quarter-disk in the z-plane *under* the mapping $f(z)$

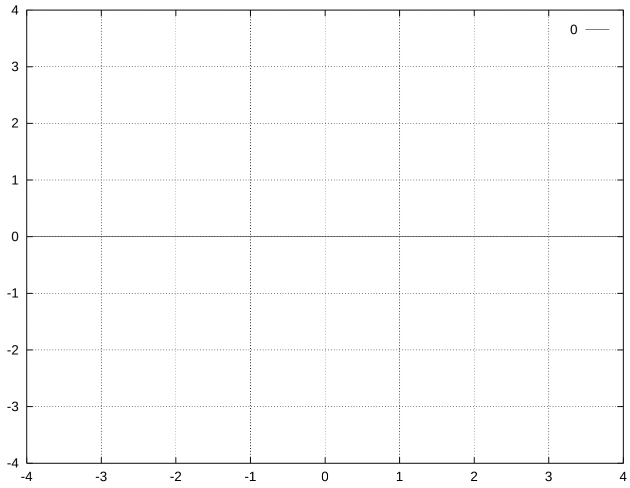
1. Write down a definition of the "quarter disk of radius 2" using complex nequalities
2. Shade in this region on your (x, y) axes (z-plane) below
3. Shade in the mapped region on your (u, v) axes (w-plane) below





On the other set of axes, sketch what the image of the mapping of the unit "quarter-disk" under $f(z) = 2z^4 - 2 - i$ looks like in the w-plane.

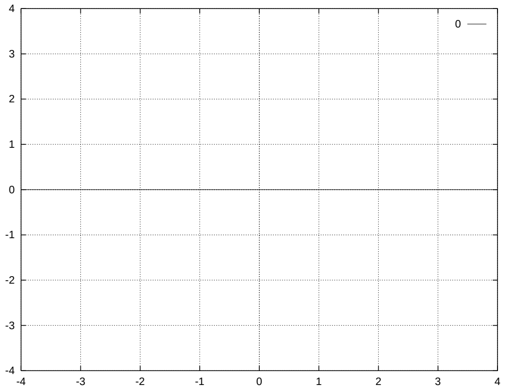


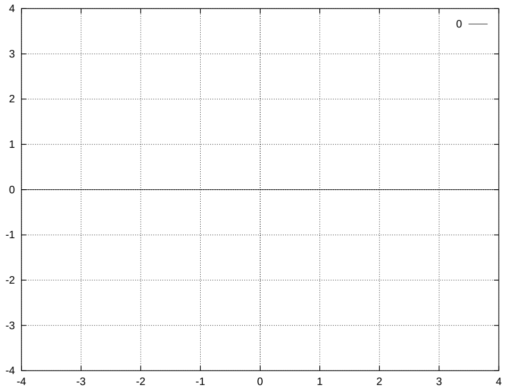


We generally can decompose mappings into 3 dominant characteristics or components. That is, mappings can be described as some combination of **rotation**, **translation** and **reflection**.

Rotation

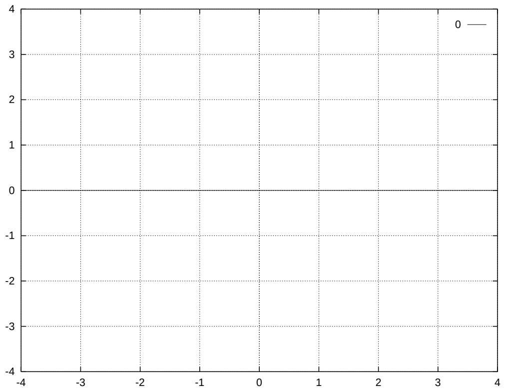
Consider $f(z) = (1+i)z$. How does this function represent a rotation mapping? (You may want to consider its effect on the set of points Im $z = 0$.)

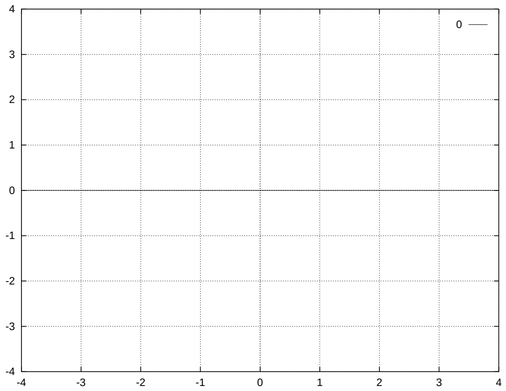




Reflection

Consider $f(z) = \overline{z}$. How does this function represent a reflection mapping? (You may want consider the effect of f on the set of points Im $z = 2$.)





Translation

Consider $f(z) = 2z - i$. How does this function represent a translation mapping? (You may want consider the effect of f on the set of points $|z| = 1$.)