Complex Analysis

Math 312 Spring 1998 — MWF 10:30am - 11:25am
Buckmire — Fowler 112

Class #5 (Friday 01/23/98)

SUMMARY Point Sets in the Complex Plane
CURRENT READING Brown & Churchill, pages 23-25
NEXT READING Brown & Churchill, pages 26-33

Any collection of points in the complex plane is called a *two-dimensional* point set, and each point is called a *member* or *element* of the set. Here are some fundamental definitions describing point sets.

Definitions

NEIGHBORHOOD
A *delta* or δ *neighborhood* of a point z_0 is the set of all points z such that $|z - z_0| < \delta$ where δ is any given positive (real) number.

DELETED NEIGHBORHOOD
A *deleted* δ *neighborhood* of z_0 is a neighborhood of z_0 in which the point z_0 is omitted, i.e. $0 < |z - z_0| < \delta$

LIMIT POINT
A point z_0 is called a *limit point, cluster point* or a *point of accumulation* of a point set S if every deleted δ neighborhood of z_0 contains points of S. Since δ can be any positive number, it follows that S must have infinitely many points. Note that z_0 may or may not belong to the set S.

INTERIOR POINT
A point z_0 is called an *interior point* of a set S if we can find a neighborhood of z_0 all of whose points belong to S.

BOUNDARY POINT
If every δ neighborhood of z_0 conrains points belonging to S and also points not belonging to S, then z_0 is called a *boundary point*.

EXTERIOR POINT
If a point is not a an interior point or a boundary point of S then it is called an *exterior point* of S.

OPEN SET
An *open set* is a set which consists only of interior points. For example, the set of points $|z| < 1$ is an open set.

CLOSED SET
A set S is said to be closed if every limit point of S belongs to S, i.e. if S contains all of its limit points. For example, the set of all points z such that $|z| \leq 1$ is a closed set.

BOUNDED SET
A set S is called *bounded* if we can find a constant M such that $z < M$ for every point in S. An *unbounded set* is one which is not bounded. A set which is both closed and bounded is sometimes called *compact*.

CONNECTED SET
An open set S is said to be *connected* if any two points of the set can be joined by a path consisting of straight line segments (i.e. a *polygonal* path) all points which are in S.

DOMAIN or OPEN REGION
An open connected set is called an *open region* or *domain*.

CLOSURE
If to a set S we add all the limit points of S, the new set is called the *closure* of S and is a closed set.
CLOSED REGION
The closure of an open region or domain is called a *closed region*.
REGION
If to an open region we add some, all or none of its limit points we obtain a set called a *region*. If all the limit points are added the region is *closed*; if none are added the region is *open*. Usually if the word *region* is used without qualifying it with an adjective, it is referring to an *open region* or *domain*.

Examples
Consider the following point sets and, using as many of the previous definitions as you can, fully describe these examples.