Complex Analysis

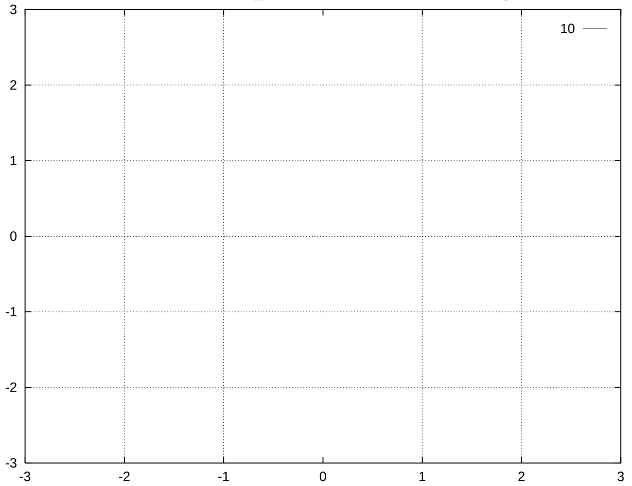
Math 312 Spring 1998 | MWF 10:30am - 11:25am
Buckmire | Fowler 112

Worksheet #2 (Wednesday 01/14/98)

SUMMARY Graphical Representations of Complex Numbers and Inequalities

READING Churchill & Brown, pages 6-11

Consider 2 complex numbers $z_1 = 3 + 0.5i$ and $z_2 = 1 + 2i$
Draw an *Argand diagram* depicting these two complex numbers in the complex plane. What physical quantity do $|z_1|$ and $|z_2|$ represent in the diagram?



Then draw in vectors that represent the complex numbers $z_1 + z_2$ and $z_1 - z_2$
Indicate what the value of $|z_1 + z_2|$ is. If I had two points at (3,0.5) and (1,2) what would the distance between these two points be?

Exercise
Consider the equation $|z - 2 + i| = 2$.
What curve does this equation represent in the complex plane?

Consider the equation $2 = \text{Re}\,(\bar{z} - i)$
What curve does this equation represent in the complex plane?

Sketch the set of points which solve these equations on the grid provided.

Complex Inequalities

There are a number of interesting inequalities that one can prove using complex variables which have significance in other arenas. The most famous of these is the *Triangle Inequality*

Triangle Inequality

$$|z_1 + z_2| \leq |z_1| + |z_2| \tag{1}$$

Can you express a geometric or vector interpretation of the Triangle Inequality?

This expression (1) is also true in a more general form known as the *Cauchy-Schwarz Inequality*

Cauchy-Schwarz Inequality

$$|z_1 + z_2 + z_3 + \ldots + z_n| \leq |z_1| + |z_2| + |z_3| + \ldots + |z_n| \tag{2}$$

$$\left|\sum_{k=1}^{n} z_k\right| \leq \sum_{k=1}^{n} |z_k| \tag{3}$$

The Cauchy-Schwarz Inequality can be proved by *mathematical induction*.

Some Other Inequalities

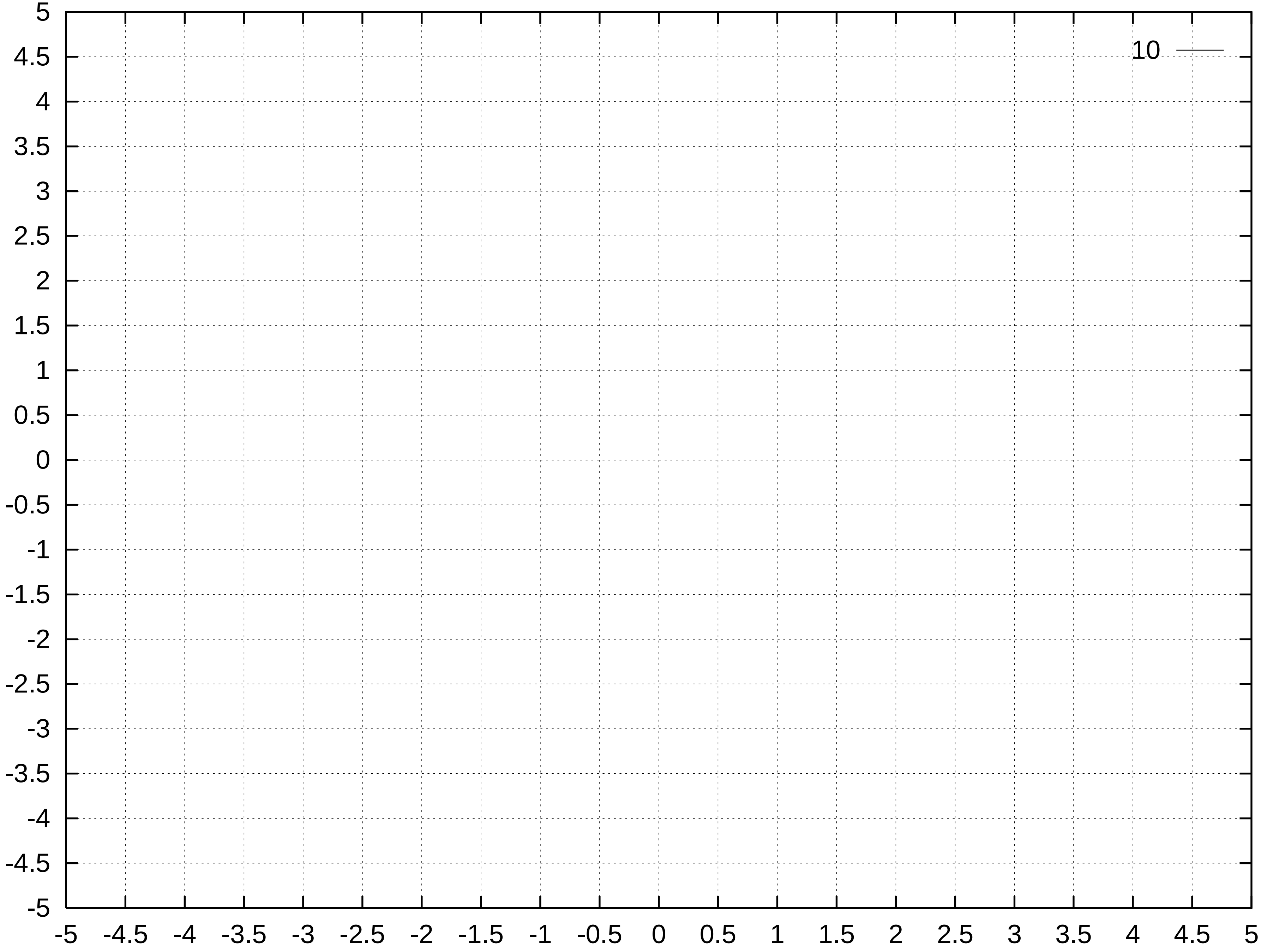
For the inequalities below, try to come up with a geometric interpretation and prove that they are true.

$$\text{Re } z \leq |\text{ Re } z| \leq |z| \text{ and } \text{Im } z \leq |\text{ Im } z| \leq |z|$$

$$|z_1 + z_2| \geq ||z_1| - |z_2||$$

$$|z_1 - z_2| \leq |z_1| + |z_2|$$

$$|z_1 - z_2| \geq ||z_1| - |z_2||$$



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5 -4.5 -4 -3.5 -3 -2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
10