

# Math 312 Spring 98

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## Quiz 4

## Complex Analysis

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Time Begun: \_\_\_\_\_

Time Ended: \_\_\_\_\_

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**Friday February 6**

**Ron Buckmire**

### **Topic :** Harmonic Functions

The point of this quiz is to show you the usefulness of the Cauchy-Riemann equations and Harmonic Functions

### **Instructions:**

1. Once you open the quiz, you have as much time as you need to complete it, but record your start time and end time at the top of this sheet.
2. You may use the book or any of your class notes. You must work alone.
3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.
4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.
5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.
6. Sometime on Saturday I will post a hint on solving this quiz on the Complex Analysis wwwboard at <http://abacus.oxy.edu/wwwboard/complex>. You can access the board by using the login and password `complex`. If you do not understand the hint or have any other questions you should post a response on the wwwboard.
7. Relax and enjoy...
8. **This quiz is due on Monday, February 9, in class. NO LATE QUIZZES WILL BE ACCEPTED.**

**Pledge:** I, \_\_\_\_\_, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

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We want to find a formula for an **entire** function  $f(z)$  but all we know is that its real part is given by  $x^3 - 3xy^2 - 4xy + 6$  and that it maps the point  $(1, 1)$  to the origin.

(a) (6 points) Use the Cauchy-Riemann Equations to find the imaginary part of  $f(z)$ , sometimes written as  $v(x, y)$ , *exactly*.

(b) (2 points) Show that both  $v(x, y)$  and its harmonic conjugate solve the 2-dimensional Laplace Equation.

(c) (2 points) What is the image of the origin of the  $z$ -plane under the mapping  $w = f(z)$ ?

**BONUS** (5 points) Write down the functions  $f(z)$  and  $f'(z)$  in a form which indicates it is a function of the complex variable  $z$  only.