Complex Analysis

Math 312 Spring 1998 — MWF 10:30am - 11:25am

Buckmire — Fowler 112

Homework Set 3

ASSIGNED: Fri Feb 13 1998 **DUE: Fri Feb 20 1998**

1. Find all values of z such that

   (a) $e^z = -2$

   (b) $\exp(2z - 1) = 1$

   (c) $\cos z = i \sin z$

   (d) $e^{iz} = 3$

   (e) $\cos z = \sqrt{2}$

2. Write the following numbers in rectangular form $a + bi$

   (a) $\exp\left(2 + \frac{i\pi}{4}\right)$

   (b) $\sin(2i)$

   (c) $\cos(1 - i)$

   (d) $\sinh(1 + \pi i)$

   (e) $\cosh\left(\frac{i\pi}{2}\right)$

3. Show that $\sin^2 z + \cos^2 z = 1$ for all complex numbers z

4. Evaluate the following

   (a) $\mathcal{L}_{\pi/2}(-1)$

   (b) $\mathcal{L}_{-\pi/2}(-1)$

   (c) The principal values of $e^{\sqrt{i}}$

   (d) The principal value of i^{i^i}

5. Find dw/dz for each of the following:

   (a) $w = \exp(\pi z^2)$

   (b) $w = \cos(2z) + i \sin(\frac{1}{z})$

   (c) $w = \exp[\sin(2z)]$

   (d) $w = \tan^3 z$

6. Explain why the function $\text{Re}\, f(z)$ is harmonic in the entire plane when $f(z) = \sin(z^2) + e^{-z} + iz + 2$

7. Find a function $\phi(x, y)$ that is harmonic in the region of the first quadrant between the curves $xy = 2$ and $xy = 4$ and takes the value 1 on the lower edge and the value 3 on the upper edge. (HINT: Remember problem 12 from Homework Set 2.)

8. Let $f(z) = i^z$.

   (a) Compute $f'(z)$.

   (b) Is $f(z)$ analytic at $z = 0$? If so, what is $f'(0)$? (What branch of log are you using?)

   (c) Compute $f'(i)$ (What branch of log are you using?)

   (d) Compute $f'(-1)$ (What branch of log are you using?)

9. Let $g(z) = z^i$.

   (a) Compute $g'(z)$. (What branch of log are you using?)

   (b) Is $g(z)$ analytic at $z = 0$? If so, what is $g'(0)$? (What branch of log are you using?)

   (c) Compute $g'(i)$ (What branch of log are you using?)

   (d) Compute $g'(-1)$ (What branch of log are you using?)

10. Show that $z^0 \equiv 1$, for all complex numbers z

11. Is $1^z \equiv 1$ for all complex numbers z?

12. Consider $g(z) = \text{ Log } (\text{ Log } (z))$

    (a) Show that $g(z)$ is analytic in the domain $\mathcal{C} \setminus \{ \text{ Im } z = 0 \cap \text{ Re } z \leq 1\}$ (the entire complex plane except for the x-axis from $x \leq 1$).

    (b) Compute $g'(z)$ within your domain of analyticity of part (a)

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(*5 points*) Use a separate sheet of paper to write down which of the problems on this assignment you had the most difficulty solving. Describe your difficulty and explain how you were able to overcome this difficulty to solve the problem. What did you learn from this process? Write atleast 3 paragraphs.

NOTES

All homework sets will be due in class one week from the class they are given out in. You are strongly encouraged to work collaboratively on the homework, though each person must hand in indvidually-written work.