Complex Analysis

Math 312 Spring 1998 — MWF 10:30am - 11:25am

Buckmire — Fowler 112

Homework Set 1

ASSIGNED: Fri Jan 23 — **DUE: Fri Jan 30**

1. Which of the points i, $2-i$ and -3 is farthest from the origin? Which is closest?
2. Draw an Argand diagram containing the following vectors
   (a) $7\text{cis}(\frac{3\pi}{4})$
   (b) $4\text{cis}(\frac{-\pi}{6})$
3. Find the argument of the following complex numbers and write in polar form.
   (a) $-1/2$
   (b) $-\pi i$
   (c) $(\sqrt{3}-i)^2$
4. Show that, for arbitrary z_1, z_2 and z_3, the following is true: $\overline{z_1 z_2 \overline{z_3}} = \overline{z_1 z_2} z_3$
5. Find a counterexample which proves that the "theorem" $\text{Arg}\,(z_1 z_2) = \text{Arg}\, z_1 + \text{Arg}\, z_2$ is **false**. (You may also want to think about why the "theorem" becomes true when Arg is replaced with arg.) Show that your counterexample is just another example that $\arg\,(z_1 z_2) = \arg z_1 + \arg z_2$ is **true** for all z.
6. Compute the following
   (a) $|(3+4i)(1+2i)(i)|$
   (b) $\left|\frac{i(2+i)^3}{(1-i)^2}\right|$
   (c) $\left|\frac{(\pi+i)^{100}}{(\pi-i)^{100}}|\right|$
7. Compute the following roots and state which is the principal root (i.e. $k=0$) in each case.
   (a) $(-16)^{1/4}$
   (b) $\sqrt{i-1}$
   (c) $(1-i\sqrt{3})^{1/3}$
8. Find the four roots of the equation $z^4+4=0$ and use them to factor z^4+4 into two quadratic factors with purely real coefficients.

9. Solve the following equations

   (a) $\dfrac{z}{1-z} = 1 - 5i$

   (b) $iz = 4 - zi$

   (c) $8z^2 + (2-i)z = 0$

   (d) $z^2 + 2z + (1-i) = 0$

10. Find z_1 and z_2 and write the answers in *exponential form*

$$\begin{aligned} (1-i)z_1 + 3z_2 &= 2 - 3i \\ iz_1 + (1+2i)z_2 &= 1 \end{aligned}$$

11. Find all complex numbers z such that $(z+1)^5 = z^5$.

12. Give sketches representing the following regions in the complex plane

   (a) $|z - 2 + i| \leq 1$

   (b) $|2z + 3| > 4$

   (c) $\text{Im } z = 1$

   (d) $|z - 4| \geq |z|$

   (e) $(\text{Re } z)^2 > 1$

13. Using your answers to the previous question, answer the following.

   (a) Which of the sets are *open*?

   (b) Which of the sets are *bounded*?

   (c) Which of the sets are *domains*?

   (d) Describe the boundary of each set.

NOTES

All homework sets will be due in class one week from the class they are given out in. You are strongly encouraged to work collaboratively on the homework, though each person must hand in indvidually-written work.