Complex Analysis

Math 312 — MWF 10:30-11:25am
Spring 1996 — 201 Fowler

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Worksheet #28

SUMMARY Cauchy Integral Formula and Integral Examples

READ pages 119-124 and 113-119 (Sections 39 and 40-44)

Cauchy's Integral Formula

Let Γ be a simple closed (positively oriented) contour. If f is analytic in some simply connected domain D containing Γ and z_0 is **any point inside** of Γ, then

$$f(z_0) = \frac{1}{2\pi i}\int_\Gamma \frac{f(z)}{z - z_0}dz$$

This leads to some of the most astonishing results in complex analysis. It is a truly amazing idea; that the value of an analytic function at a point z_0 in a simply-connected domain depends on values it takes on some closed contour Γ encircling the point. There are over a *dozen* theorems which arise directly from the **C.I.F.** , as it is known.
(see the extra sheet summarizing nine of them)

The proof is reasonably straightforward and involves the continuity of $f(z)$ at every point in D and the formula for bounding a contour integral.

Pregnant Pause: Take 5 minutes and think about the proof. To illustrate that you really DO understand it, try and explain the proof to your nearest neighbor.

Here is the first of many amazing ideas derived from the **CIF**.
Let Γ be a simple closed (positively oriented) contour. If f is analytic in some simply connected domain D containing Γ and z_0 is **any point inside** of Γ, then

$$f^{(n)}(z_0) = \frac{1}{2\pi i}\int_\Gamma \frac{f(z)}{(z - z_0)^{n+1}}dz$$

Example

$$\int_{|z|=1} \frac{z^2 e^{z^2}}{(2z+i)(z+2)}\,dz =$$

Let's summarize what we know about integrals by trying to do the following examples. For each one, use the contours displayed on the right-hand side of each integral and then state what method you would use to evaluate it. For some of them you should be able to immediately write down the value of the integral.

$\oint_C \overline{z}\, dz$

$\oint_C \frac{x^2 - y^2}{2} + xyi\, dz$

$\oint_C \frac{dz}{(z-3)^4}$

$\oint_C \frac{dz}{z^2 + \pi^2}$

$\oint_C \frac{\sinh(2z)}{z^2 + \pi^2}\, dz$

QUESTION OF THE DAY (Due: 4/17)

Evaluate the following integral

$$\oint_C \frac{z+i}{z^3 + 2z^2} dz$$

where the contour C is traversed *twice* in the clockwise direction.

a the circle $|z| = 1$ traversed once counterclockwise

b the circle $|z + 2 - i| = 2$ traversed once counterclockwise

c the circle $|z - 2i| = 1$ traversed once counterclockwise