Complex Analysis

Math 312 — MWF 10:30-11:25am

Spring 1996 — 201 Fowler

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Worksheet #25

SUMMARY More Complex Integration and Antiderivatives

READ pages 95-108 (Sections 33-35)

Previously we learned how to integrate functions of a complex variable. Let's do some more examples.

Example

Compute $\int_C 2\overline{z}^2\, dz$ where C is a directed line segment from $z = 2$ to $z = -2$

1: write down a parametrization for C, $z(t)$

2: Convert the contour integral into an integral in t variables

3: Integrate!

Repeat $\int_C 2\overline{z}^2\, dz$, this time using C being a circular arc from $z = 2$ to $z = -2$

Properties of Contour Integrals

$$\int_C f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz$$

$$\int_{-C} f(z)dz = -\int_C f(z)dz$$

$$\int_C z_0 f(z)dz = z_0 \int_C f(z)dz, \qquad z_0 \in \mathbf{C}$$

$$\int_C f(z) + g(z)dz = \int_C f(z)dz + \int_C g(z)dz$$

$$\left|\int_C f(z)dz\right| \leq ML$$

where L is the length of the contour and $\int_a^b |z'(t)|dt \leq L$
and M is an upper bound on $f(z)$, $|f(z)| \leq M$

Examples

If Γ is the arc of the circle $|z| = 2$ traversed in the counter-clockwise direction, then we want to show that

$$\left|\int_\Gamma \frac{e^z}{z^2+1}dz\right| \leq \frac{4\pi e^2}{3}$$

Compute $\int_C x^2 - y^2 + 3xyi\ dz$
where C is (a) a contour consisting of line segments going from 1 to i along the x-axis and y-axis or (b) a contour consisting of one line segment joining 1 to i directly

QUESTION OF THE DAY (Due: 4/8)

Without evaluating the integral, show that

$$\int_C \frac{dz}{z^2-1} \leq \frac{\pi}{3}$$

where C is the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$ that lies in the first quadrant. [HINT: you may have to use inequalities on page 10 of the text, i.e. $|z_1 - z_2| \leq |z_1| + |z_2|$ and $|z_1 - z_2| \geq |z_1| - |z_2|$]