Complex Analysis

Math 312	MWF 10:30-11:25am
Spring 1996	201 Fowler

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This worksheet is a sample of the type of problems that you should be able to do now that we have finished this unit on **Functions of a Complex Variable**. Some of topics we covered were:

- complex functions of a complex variable
- limits, limit rules
- differentiability, continuity
- Differentiation rules
- Cauchy-Riemann Equations ($u_x = v_y, u_y = -v_x$)
- Analyticity ($\Leftrightarrow$CRE + continuity of u_x, u_y, v_x, v_y)
- entire functions
- Harmonic functions, harmonic conjugates
- elementary functions, e.g. e^z, $\sin z$, $\cos z$, $\sinh z$, $\cosh z$
- $\log z$, Log z and $\mathcal{L}_\alpha$ (i.e. $\alpha <$ arg $z \leq \alpha + 2\pi$)
- Properties of logarithms e.g. $z = e^{\log z}$
- complex powers z^c, c^z
- linear transformations (rotations, translations, scalings) $w = Az + B$
- inverse mapping $w = 1/z$
- linear fractional transformations $w = (az + b)/(cz + d)$
- poles, mapping circles and lines to circles and lines
- point at infinity
- mapping by LFTs, e^z, z^2, Log z and others...
- conformal mappings are mappings by analytic functions and preserve angles

REVIEW EXAM

These problems will review these topics. If you can do them all you should excel on the test!

(1) Mapping

(a) Find the bilinear mapping which maps $(-4, 0, \infty)$ to $(\infty, -5/4, 2)$

(b) Find the fixed points of the mapping in part (a). i.e., find the point(s) which have the property that $w(z) = z$

(c) What is the image of the *exterior* of the circle ($|z| = 4$) under this mapping? What kind of shape will the boundary be? (Draw a sketch)

(d) Give an equation in $w = (u, v)$ coordinates describing the boundary in part (c)

(2) Logarithms

(a) Show that only when Re $z_1 > 0$ and Re $z_2 > 0$ is it true that

$$\text{Log } (z_1 z_2) = \text{Log } z_1 + \text{Log } z$$

(b) Let z_1 and z_2 be any two numbers that satisfy the condition in part (a) and illustrate that your formula works

(3) Analyticity

(a) Show that the function $u(x, y) = 2x(1 - y)$ is harmonic and find its *harmonic conjugate* $v(x, y)$

(b) Determine the function $f(z)$ which is composed of $u(x, y)$ and $v(x, y)$ Is $f(z)$ analytic? On what set? Why?

(c) Show that $x^2 + iy^3$ satisfy the CRE at $(0, 0)$. Does this mean $x^2 + iy^3$ is analytic? Why/why not?

(4) Branches

Use the branch of $\log z$ denoted as $\mathcal{L}_{5\pi/2}$ to define $f(z) = z^i$

(a) Compute $f(-i)$ and $f'(-i)$

(b) Let $g(z) = \exp[i\text{Log } z]$, Compute $g(-i)$ and $g'(-i)$

(c) Do you get different answers for g and f being evaluated at the same point? Why/why not?

(d) Solve the equation $f(z) = -2i$ and $g(z) = -2i$

(e) Sketch the branch cuts of the functions $f(iz)$ and $g(-z)$