Complex Analysis

Math 312 MWF 10:30-11:25am
Spring 1996 201 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ ron@oxy.edu ~ Mon 1-3pm & Thu 1-2pm

Worksheet #17

SUMMARY Branch cuts and applications of elementary functions

READ pages 207-208 (Section 64)

Branch cuts revisited
A **branch cut** is a set of points which we must remove from the domain of definition of a multi-valued function in order to construct a single-valued function.
Let us look at Log $(3z - i)$ again. First, let's consider Log (w).
Where in the w-plane is Log (w) not analytic?

What are the set of points where $\text{Re}(w) < 0$ and $\text{Im } w = 0$ if $w = 3z - i$?
Draw these points on the axes below.

Now consider Log $(z^3 - 2)$. This is a much more complicated function.
How does it differ from $\log(z^3 - 2)$ or $\mathcal{L}_\alpha(z^3 - 2)$?
What are the equations that its branch cut will satisfy?
Sketch the branch cut for Log $(z^3 - 2)$ on the axes below.

Other functions have branch cuts, too, of course. We usually have to *choose* the branch cut so that our function in question is analytic at a required point.

Examples

1: Define a branch of $f(z) = (z^2 - 1)^{1/2}$ so that it is analytic on the exterior of the unit circle $|z| > 1$

(a) First write down the *principal branch* of $f(z)$. Where are its branch cuts? [What equations/inequalities do you have to solve?]

(b) Sketch the branch cuts on the axes below.

(c) What if we redefine $(z^2 - 1)^{1/2} = z(1 - 1/z^2)^{1/2}$? Where are the branch cuts of the principal branch of our new function? Sketch them below

(d) Write down the function which solves our problem #1

The problem of "branch chasing" for complicated functions can be extremely tedious but it is necessary if we want them to be analytic. Thankfully, it is seldom necessary for our elementary applications.