Complex Analysis

Math 312 — MWF 10:30-11:25am
Spring 1996 — 201 Fowler

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Worksheet #16

SUMMARY Complex Exponents z^c and c^z

READ pages 80-84 (Section 28.)

If $z \neq 0$ and $c \in \mathcal{C}$, the function z^c is defined as

$$z^c = \exp(c \log z)$$

Since $\log z$ is a multi-valued function, z^c will have multiple values. How many values depends on the nature of c.

$$z^c = \begin{cases} z^{n/m} & \text{if } c \text{ is rational, i.e. } n/m & \text{finite number of values (m)} \\ z^n & \text{if } c = n, \text{ where } n \text{ is an integer} & \text{single value} \\ z^c & \text{all other complex numbers} & \text{infinite number of values} \end{cases}$$

Examples
Compute the following:
(a) $(1+i)^{1-i} =$

(b) $(0.5-i)^3 =$

(c) $(-1)^{2/3} =$

Differentiating
If you choose a branch of z^c which is analytic on an open set, then

$$\frac{d}{dz}(z^c) = cz^{c-1}$$

where the branch of the log used in evaluating z^c is the same branch used in evaluating z^{c-1}
Similarly, we can define the *complex exponential function with base c*

$$c^z = \exp(z \log c)$$

If a single value of c is chosen, then c^z is an entire function such that

$$\frac{d}{dz}(c^z) = \frac{d}{dz}\exp(z \log c) = c^z \log c$$