Complex Analysis

Math 312 MWF 10:30-11:25am
Spring 1996 201 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ ron@oxy.edu ~ Mon 1-3pm & Thu 1-2pm

Worksheet #13

SUMMARY Analyticity and Harmonic Functions

Laplace's Equation

The partial differential equation shown below is known as **Laplace's Equation**.

$$\nabla^2\phi = \Delta\phi = \frac{\partial^2\phi(x,y)}{\partial x^2} + \frac{\partial^2\phi(x,y)}{\partial y^2} = 0$$

Solutions $\phi(x,y)$ which solve Laplace's equation are very important in a number of areas of mathematical physics and applied mathematics. Some of these applications are:

- electrostatic potential in two-dimensional free space
- scalar magnetostatic potential
- stream function and velocity potential in fluid flow (aerodynamics, etc)
- spatial distribution of equilibrium temperature

Harmonic Function

A real-valued function $\phi(x,y)$ is said to be **harmonic** in a domain D if all its second-order partial derivatives are continuous in D and if ϕ satisfies Laplace's Equation at each point $(x,y) \in D$.

If $f(z)$ is analytic on a domain D then both $u(x,y)$ =Re$(f(z))$ and $v(x,y)$ =Im$(f(z))$ are harmonic in D.

The proof follows directly from the CRE.

(Take 3 minutes and try and come up with it.)

Given a harmonic function $u(x,y)$ defined on an open connected set D we can construct a **harmonic conjugate** $v(x,y)$ so that the combined function $f = u(x,y) + iv(x,y)$ will be analytic on the domain D.

Example

Given $u(x,y) = x^3 - 3xy^2 + y$ construct an analytic function $f(z)$

By studying harmonic functions we can learn about analytic functions, and vice-versa.