Complex Analysis

Math 312 — MWF 10:30-11:25am
Spring 1996 — 201 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ `ron@oxy.edu` ~ Mon 1-3pm & Thu 1-2pm

Worksheet #9

SUMMARY Limits of Complex Functions

Limits

The rules on limits of complex functions are identical to the rules for real functions of two variables (as you'd expect)

Suppose that

$$\lim_{z \to z_0} f(z) = w_0 \text{ and } \lim_{z \to z_0} F(z) = W_0$$

then

$$\begin{aligned}
\lim_{z \to z_0} [f(z) + F(z)] &= w_0 + W_0 \\
\lim_{z \to z_0} [f(z)F(z)] &= w_0 W_0 \\
\lim_{z \to z_0} \frac{f(z)}{F(z)} &= \frac{w_0}{W_0} \qquad (W_0 \neq 0) \\
\lim_{z \to z_0} |f(z)| &= |w_0| \\
\lim_{z \to z_0} c &= c \\
\lim_{z \to z_0} z^n &= z_0^n \\
\lim_{z \to z_0} P(z) &= P(z_0) \qquad \text{(where } P(z) \text{ is a polynomial)}
\end{aligned}$$

Suppose that

$$f(z) = u(x,y) + iv(x,y), \qquad z_0 = x_0 + iy_0 \qquad \text{and } w_0 = u_0 + iv_0$$

THEN

$$\lim_{z \to z_0} f(z) = w_0$$

if and only if

$$\lim_{(x,y) \to (x_0,y_0)} u(x,y) = u_0 \text{ and } \lim_{(x,y) \to (x_0,y_0)} v(x,y) = v_0$$

Examples

Compute

$$\lim_{z \to 1+2i} 2|z| + iz + 2.5 - .1i =$$

$$\lim_{z \to 3\pi i} \text{Re } e^z =$$

$$\lim_{z \to 0} \frac{z^8 + z^4 + z^2 + z - 1}{z^3 + 4z^3 - 9} =$$