Complex Analysis

Math 312 — MWF 10:30-11:25am
Spring 1996 — 201 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ `ron@oxy.edu` ~ Mon 1-3pm & Thu 1-2pm

---

Worksheet #6 **(Monday 02/05/96)**

SUMMARY square roots of complex numbers; algebraic/transcendental numbers

The m *m-th roots of unity* are given by

$$\exp\left(\frac{2\pi i k}{m}\right), \quad k = 0, 1, \ldots, m-1$$

The **primitive m-th root of unity**, ω_m is defined as $\omega_m = \exp\left(\frac{2\pi i}{m}\right)$
The m-th root of a complex number z_0 can be written as

$$|z_0|^{1/m} \exp\left(\frac{i(\theta_0 + 2\pi k)}{m}\right), \qquad k = 0, 1, \ldots, m-1, \quad \text{where } \theta = \arg z$$

The **principal m-th root** of a complex number is the complex number one obtains when $k = 0$ and $\theta_0 = \text{Arg } z_0$ in the above formula, i.e. $|z_0| \exp[\frac{i\theta_0}{m}]$

Example
Compute $\sqrt{5 - 12i}$.

Algebraic/Transcendental Numbers
A number z is called *algebraic* if it is a solution of a polynomial equation $a_0 z^n + a_1 z^{n-1} + \ldots + a_{n-1} z + a_n = 0$ where $\{a_i\}$ are integers (or reals or rationals). Numbers which are not algebraic, are called *transcendental numbers*.

Can you think of a transcendental number? Write down an example.
Can you think of an algebraic number? Write down an example.

Is $\sqrt{3} + \sqrt{2}$ a transcendental number? If not, what polynomial equation does it solve?