Complex Analysis

Math 312 — MWF 10:30-11:25am
Spring 1996 — 201 Fowler

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Homework Set 4

ASSIGNED: Mon Mar 11 — **DUE: Wed Mar 27**

1. On page 78 in the text *Churchill & Brown* do: **1, 3, 5, 7**
2. On page 83 in the text *Churchill & Brown* do: **1, 2, 3, 4**
3. Let $f(z) = i^z$.
   (a) Compute $f'(z)$. (What branch of log are you using?)
   (b) Is $f(z)$ analytic at $z = 0$? If so, what is $f'(0)$? (What branch of log are you using?)
   (c) Compute $f'(i)$ (What branch of log are you using?)
   (d) Compute $f'(-1)$ (What branch of log are you using?)
4. Let $g(z) = z^i$.
   (a) Compute $g'(z)$. (What branch of log are you using?)
   (b) Is $g(z)$ analytic at $z = 0$? If so, what is $g'(0)$? (What branch of log are you using?)
   (c) Compute $g'(i)$ (What branch of log are you using?)
   (d) Compute $g'(-1)$ (What branch of log are you using?)
5. Show that $z^0 \equiv 1$, for all complex numbers z
6. Is $1^z \equiv 1$ for all complex numbers z?
7. Find a branch of $\sqrt{z^2 - 1}$ which is analytic in the *interior* of the unit circle $|z| < 1$.
8. Find a linear transformation mapping the circle $|z| = 1$ onto the circle $|w - 5| = 3$
9. Map $|z - 2| = 1$ under the following mappings (sketch the results)
   (a) $w = z - 2i$
   (b) $w = 3iz$
   (c) $w = \dfrac{z-2}{z-1}$
   (d) $w = \dfrac{z-4}{z-3}$
   (e) $w = \dfrac{1}{z}$

10. On page 219 in the text *Churchill & Brown* do: **1, 2, 6, 7**

11. Find the Möbius transformation which maps $0, 1, \infty$ to the following respective points:

(a) $0, i, \infty$

(b) $0, 1, 2$

(c) $-i, \infty, 1$

(d) $-1, \infty, 1$

12. What is the image of the third quadrant under $w = \dfrac{z+i}{z-i}$?

13. Find an LFT which takes the half-plane $x - y < 1$ to unit disk $|w| < 1$ (See sketch below)

BONUS:

Map the shaded region below conformally onto the interior of the unit circle $|w| < 1$ [HINT: Use an LFT which maps 2 to ∞ and converts the shaded area to a strip. Then use the exponential mapping to map that to the upper-half plane. Then map the upper half-plane to the interior of the unit circle]

<u>NOTE:</u> Exam 2 on **Monday April 1** will cover the topics in Homework Sets 3 and 4.