Class 20: **Monday March 28**

TITLE Implications of the Cauchy-Goursat Theorem
CURRENT READING Zill & Shanahan, §5.4-5.5
HOMEWORK SET #9 (DUE WED MAR 30)
Zill & Shanahan, §5.3 2, 9, 12, 20, 25, 27. **23*,29***. §5.4 1, 8, 18, 22. **25***. §5.5 7, 22, 23, 24.

SUMMARY
We shall consider the awesome implications of the Cauchy-Goursat Theorem.

RECALL
Previously we learned that the **Cauchy-Goursat Theorem** implies that,
given f is continuous in an open connected set D if any one of the following statements is true, then so are the other two:

(a) f has an antiderivative in D, called $F(z)$, such that $F'(z) = f(z)$

(b) the integrals of $f(z)$ along **all** contours lying entirely in D extending from z_1 to z_2 all have the same value, namely $F(z_2) - F(z_1)$

(c) the integrals of $f(z)$ around closed contours lying entirely in D all have value zero

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| EXAMPLE |
Evaluate the following integrals. In each case sketch the contour you used and explain how you evaluated the integral, i.e. what idea are you using. Also answer the questions.

1. $\displaystyle\int_{-i}^{i} \frac{dz}{z}$

Does the integral value change depending on the path from $-i$ to i?
Does $1/z$ have an antiderivative? In what domain?
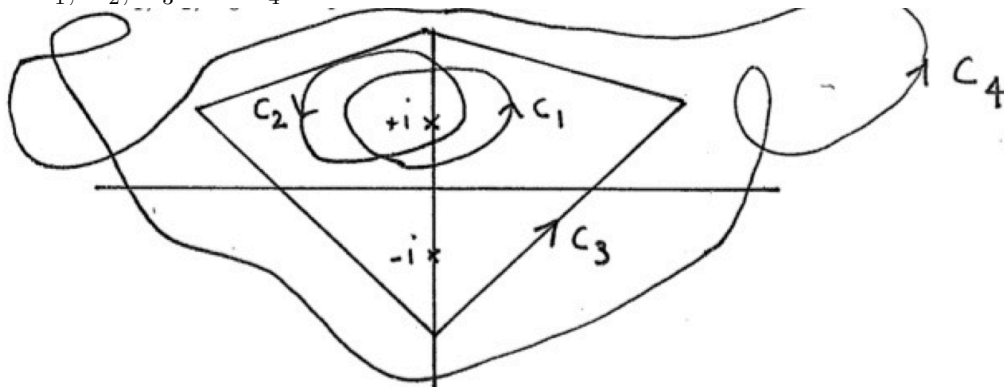
2. $\displaystyle\oint_{|z|=1} \frac{dz}{z}$

Does the integral value change depending on what direction one takes along $|z| = 1$?
What would the value of the integral be if the contour were traversed twice in the clockwise direction?

GROUPWORK

Consider the following integrals of the function $f(z) = \dfrac{1}{z^3 + z}$

$A = \oint_{C_1} f(z)\,dz$, $B = \oint_{C_2} f(z)\,dz$, $C = \oint_{C_3} f(z)\,dz$ and $D = \oint_{C_4} f(z)\,dz$

The contours C_1, C_2, C_3 and C_4 are as sketched below:

Which of the following equations are true? Give reasons for your answers.

1. $A = B$?

2. $B = C$?

3. $C = D$?

4. $D = A$?

EXAMPLE

Using partial fractions, we can write $\dfrac{1}{z^3 + z} = \dfrac{P}{z + i} + \dfrac{Q}{z - i} + \dfrac{R}{z}$

Find P, Q and R

Evaluate A, B, C and D

Exercise

Write down, in your own words, a sentence describing each one of the following theorems. You may also want to write down symbols, pictures or even integrals which help you to understand these theorems.

Remember theorems have CONDITIONS and CONCLUSIONS. What are the conditions and conclusions for each of the following theorems?

Cauchy-Goursat Theorem

Path Independence Theorem or **Fundamental Theorem of Contour Integration**

Deformation Invariance Theorem