Complex Analysis

Math 312 Spring 2016 | Fowler 309 MWF 11:45am-12:40pm

 | http://sites.oxy.edu/ron/math/312/16/

Class 10: **Wednesday February 10**

TITLE Differentiability of Complex Functions
CURRENT READING Zill & Shanahan, Section 3.2
HOMEWORK SET #4 (DUE WED FEB 17)
Zill & Shanahan, Chap 2 Review 1-10, §3.1.1: #2, 11, 17, **20***; §3.1.2: #28, 31, 37, **50***;

SUMMARY
We shall move on from our discussion of continuity to a discussion of differentiability for a complex function of a complex variable. This will lead us to the idea of **analyticity** and the famous Cauchy-Riemann Equations.

Definition of the Derivative

Let f be defined in a neighborhood around z_0. The **derivative** of f at z_0, denoted by $f'(z_0)$, is defined by

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

provided the above limit exists. The function f is said to be *differentiable* at z_0.
Consider $f(z) = z^2$. Write down the expression $\frac{\Delta w}{\Delta z} = \frac{f(z+\Delta z) - f(z)}{\Delta z}$

The derivative $\dfrac{dw}{dz} = f'(z)$ is defined as $f'(z) = \displaystyle\lim_{\Delta z \to 0} \frac{\Delta w}{\Delta z}$
Evaluate this limit for our function $f(z) = z^2$.

Write down $f'(z)$

Write down the real and imaginary parts of the function $f(z) = z^2$

Write down the real and imaginary parts of the function $f'(z)$ See any patterns between the real and imaginary parts of $f(z)$ and $f'(z)$?

Rules of Differentiation

The standard rules of differentiating function that you learned for real functions basically apply to complex functions. Namely:

$$\frac{d}{dz}(c) = 0 \qquad \frac{d}{dz}(z) = 1 \qquad \frac{d}{dz}(z^n) = nz^{n-1} \qquad \frac{d}{dz}(e^z) = e^z$$

Linearity

$$\frac{d}{dz}[cf(z) + g(z)] = cf'(z) + g'(z) \qquad c \text{ constant}$$

Product Rule

$$\frac{d}{dz}[f(z)g(z)] = f'(z)g(z) + f(z)g'(z)$$

Quotient Rule

$$\frac{d}{dz}\left[\frac{f(z)}{g(z)}\right] = \frac{f'(z)g(z) - f(z)g'(z)}{(g(z))^2}$$

Aspects of Differentiation

One of the most important aspects to remember about differentiability and continuity is:

DIFFERENTIABILITY $\Rightarrow$ CONTINUITY

CONTINUITY DOES NOT IMPLY DIFFERENTIABILITY.

$\boxed{\text{GroupWork}}$

Given $g(z) = z^2 + z + i$ and $f(z) = \dfrac{1}{z}$

$g'(z) =$

$f'(z) =$

$[g(z)f(z)]' =$

$[g(z)/f(z)]' =$