Complex Analysis

Math 312 Spring 2016 | Fowler 309 MWF 11:45am-12:40pm

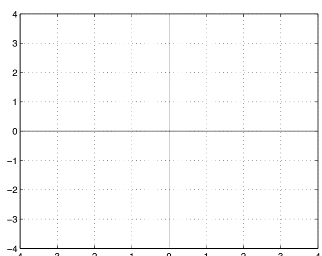
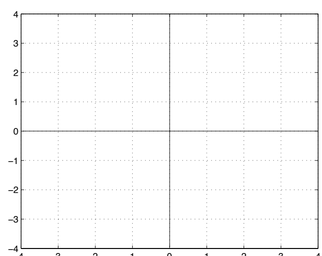
 | http://sites.oxy.edu/ron/math/312/16/

Class 7: Wednesday February 3

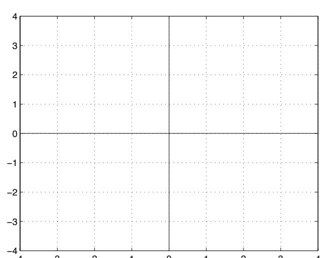
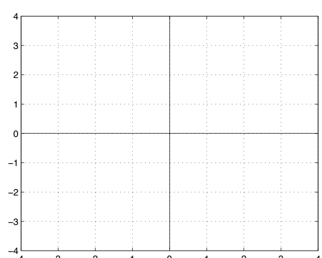
TITLE Graphical Interpretation of Complex Linear Functions
CURRENT READING Zill & Shanahan, Section 2.3
HOMEWORK Zill & Shanahan, §2.3 9, 18, 19, 34, **29***.

SUMMARY
We shall focus on the graphical interpretations of the mapping $f(z) = az + b$. We generally can decompose linear complex mappings into 3 dominant characteristics or components. That is, mappings can be described as a combination of **rotation**, **translation** and **magnification**.

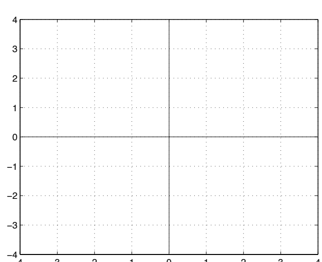
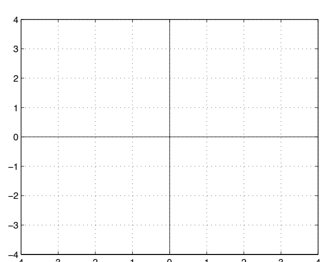
Rotation
Consider $R(z) = iz$. How does this function represent a rotation mapping?
(Consider its effect on the set of points $\{z \in \mathbb{C} : \text{ Im } z = 0\}$.)

Scaling
Consider $S(z) = 2z$. How does this function represent a scaling mapping?
(Consider the effect of S on the set of points $\{z \in \mathbb{C} : |z| = 1\}$.)

Translation
Consider $T(z) = z - i + 2$. How does this function represent a translation mapping?
(Consider the effect of T on the set of points $\{z \in \mathbb{C} : |z| = 1\}$.)

Complex Linear Function as Mapping

The complex linear function $f(z) = az + b$ can be written as

$$w = f(z) = az + b = |a|\left(\frac{a}{|a|}z\right) + b$$

which is the composition of the mappings, $f_3(z) = \left(\frac{a}{|a|}\right)z$, $f_2(z) = |a|z$ and $f_1(z) = z + b$.

EXAMPLE 1

We can show that $f(z) = az + b = f_1(f_2(f_3(z))) = (f_1 \circ (f_2 \circ f_3))(z)$

Exercise 1

In words, describe each of the following mappings.

$f_1(z) = z + b$ is

$f_2(z) = |a|z$ is

$f_3(z) = \left(\frac{a}{|a|}\right)z$ is

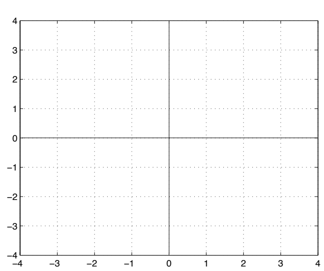
In order of application, the linear mapping $f(z) = az + b$ is a ____________________ followed by a ____________________ followed by a ____________________.

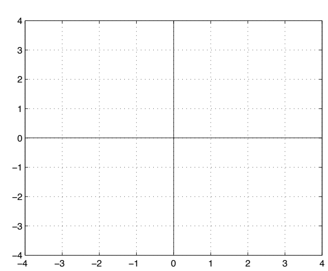
Reflection Mapping Is Conjugation

Another familiar action on point sets in the plane is the reflection mapping. However, it is **not** represented by a linear mapping.

Consider $f(z) = \overline{z}$. How does this function represent a reflection mapping?

(Consider the effect of f on the set of points $\{z \in \mathbb{C} : \text{ Im } z = 2\}$)





GroupWork

Consider the function $w = F(z) = R(S(T(z)))$ where $R(z) = iz$, $S(z) = 2z$ and $T(z) = z - i + 2$. Find the image of $\mathcal{S}$, the disc $|z| = 1$, under the action of $F(z)$ in two different ways.

1) Show the composition of three mappings as a sequence which takes $\mathcal{S}$ to the final image $\mathcal{S}'$ in the space below. (Write down in words what happens at each step, paying careful attention to what happens to the center of the circle.)

2) Show algebraically that $w = T(z) = 2iz + 2 + 4i$ and that if the pre-image is $|z| = 1$ then the image is $|w - 2 - 4i| = 2$.