
Complex Analysis

Math 312 Spring 2016

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Fowler 309 MWF 11:45am-12:40pm

<http://sites.oxy.edu/ron/math/312/16/>

Class 2: Friday January 22

TITLE Graphical Representations of Complex Numbers and Inequalities

READING Zill & Shanahan, Section 1.2

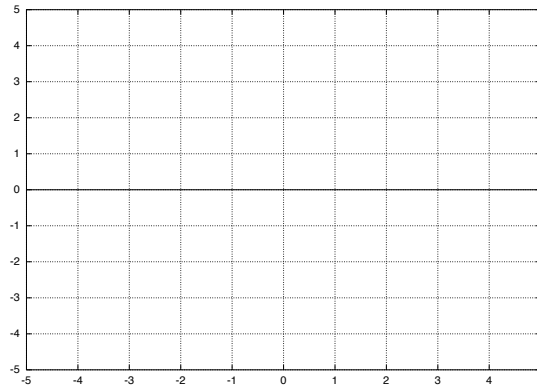
HOMEWORK Zill & Shanahan, Section 1.2 # 2, 5, 20, 29, 33, **Extra Credit: 38**

SUMMARY

We shall be introduced to the graphical (geometrical) representation of complex numbers, particularly the Argand diagram.

Consider two complex numbers $z_1 = 3 + 0.5i$ and $z_2 = -1 - 2i$

Draw an *Argand diagram* depicting these two complex numbers in the complex plane (on the grid provided). What physical quantity do $|z_1|$ and $|z_2|$ represent in the diagram?



Then draw in vectors that represent the complex numbers $z_1 + z_2$ and $z_1 - z_2$

Indicate what the value of $|z_1 + z_2|$ is. If I had two points at $(3,0.5)$ and $(-1,-2)$ what would the distance between these two points be?

GROUPWORK

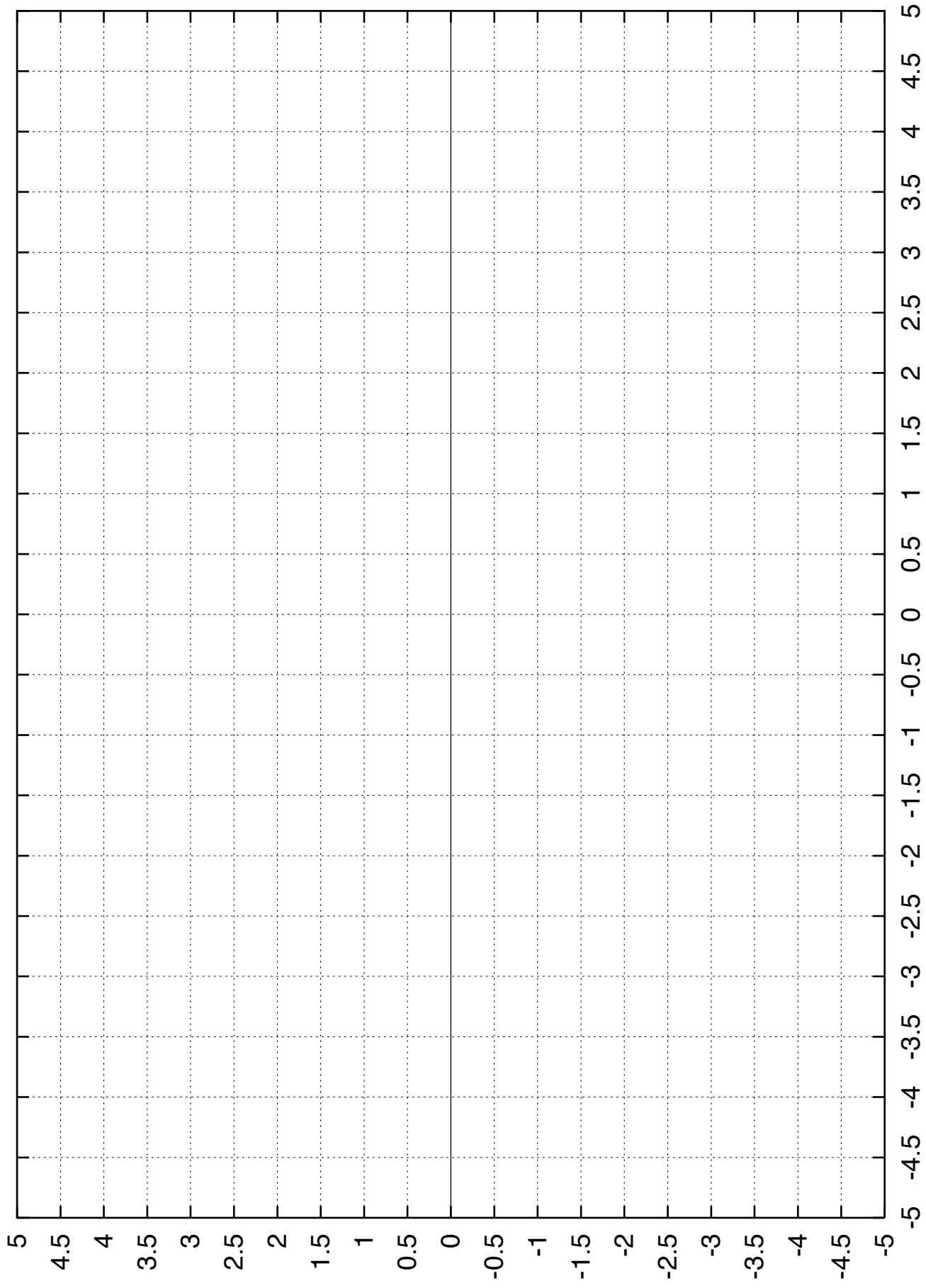
Consider the equation $|z - 2 + i| = 2$.

What curve does this equation represent in the complex plane?

Consider the equation $2 = \operatorname{Re}(\bar{z} - i)$

What curve does this equation represent in the complex plane? How about $\operatorname{Im}(\bar{z} - i) = 2$?

Sketch the set of points which satisfy each of these equations on the grid provided (on the next page)



Complex Inequalities

There are a number of interesting inequalities that one can prove using complex variables which have significance in other arenas.

The most famous of these is the *Triangle Inequality*

Triangle Inequality

$$|z_1 + z_2| \leq |z_1| + |z_2| \quad (1)$$

Can you express a geometric or vector interpretation of the Triangle Inequality?

This expression (1) is also true in a more general form known as the *Cauchy-Schwarz Inequality*

Cauchy-Schwarz Inequality

$$\begin{aligned} |z_1 + z_2 + z_3 + \dots + z_n| &\leq |z_1| + |z_2| + |z_3| + \dots + |z_n| \\ \left| \sum_{k=1}^n z_k \right| &\leq \sum_{k=1}^n |z_k| \end{aligned}$$

The Cauchy-Schwarz Inequality can be proved by *mathematical induction*.

PROOF

Some Other Inequalities

For the inequalities below, try to come up with a geometric interpretation and prove that they are true.

$$\operatorname{Re} z \leq |\operatorname{Re} z| \leq |z| \text{ and } \operatorname{Im} z \leq |\operatorname{Im} z| \leq |z|$$

$$|z_1 + z_2| \geq ||z_1| - |z_2|| \tag{2}$$

$$|z_1 - z_2| \leq |z_1| + |z_2| \tag{3}$$

$$|z_1 - z_2| \geq ||z_1| - |z_2|| \tag{4}$$

Exercise

Inequalities (2), (3) and (4) are found on page 12 of Zill & Shanahan. Pick one of them and try and to prove it is true.

EXAMPLE

Show that an upper bound for $\left| \frac{-1}{z^4 + 3z^2 + 2} \right|$ when $|z| = 2$ is $\frac{1}{6}$.