1. **(10 points)** One interesting application of contour integration is the ability to find the area of odd shapes in the plane. If we denote the area enclosed by a positively-oriented contour $C$ by $A$, then

\[ A = \frac{1}{2i} \oint_C z \, dz \]

(a) **(6 points)** Recalling that the parametrization given by $z(t) = a \cos t + ib \sin t$, $0 \leq t \leq 2\pi$ represents an elliptical contour $C$ with horizontal axis $a$ and vertical axis $b$ use the formula for $A$ to compute the area enclosed by an ellipse. (Your final answer should only involve $\pi$, $a$ and $b$.)

\[
A = \frac{1}{2i} \int_0^{2\pi} \left( (a \cos t - ib \sin t)(-a \sin t + ib \cos t) \right) dt \\
= \frac{1}{2i} \int_0^{2\pi} a^2 \cos^2 t + b^2 \sin^2 t + i (a^2 \sin^2 t + ab \cos^2 t) dt \\
= \frac{1}{2i} \int_0^{2\pi} \left( a^2 \cos^2 t + b^2 \sin^2 t + \frac{1}{2} (a^2 + b^2) \right) dt \\
= \frac{1}{2} \left( a^2 + b^2 \right) \left( \sin^2 t + \frac{1}{2} \right) \bigg|_0^{2\pi} + \pi ab \\
= \frac{1}{2} \left( a^2 + b^2 \right) (0 + \frac{1}{2}) + \pi ab \\
= \frac{1}{8} \left( a^2 + b^2 \right) + \pi ab
\]

(b) **(4 points)** On the same contour as part (a) find the value of $B$, where

\[ B = \frac{1}{2i} \oint_C z \, dz \]

(HINT: think how the functions in the integrands of $A$ and $B$ are different to obtain the value $B$ of this integral without doing very much work.) **EXPLAIN YOUR ANSWER.**

\[ z \text{ is analytic everywhere so by the Cauchy-Goursat Theorem } \oint_C z \, dz = 0 \text{ regardless of the shape of } C \]

\[ \text{We know } z \text{ is analytic because it's a polynomial in } t \text{ and all polynomials in } t \text{ are ENTIRE (analytic everywhere)} \]