Quiz 3

Name: ________________________________

Time Begun: __________________________
Time Ended: __________________________

ASSIGNED: Friday February 5
DUE: Monday February 8

Prof. Ron Buckmire

Topic: Understanding Linear Complex Mappings

The learning goal of this quiz is to provide another example of using functions as mappings and to demonstrate your understanding of the concept of function in the context of complex numbers.

Reality Check:

EXPECTED SCORE: ________/10
ACTUAL SCORE: ________/10

Instructions:

1. Once you open the quiz, you have 30 minutes to complete, please record your start time and end time at the top of this sheet.

2. You may use the book or any of your class notes. You must work alone.

3. If you use your own paper, please staple it to the quiz before coming to class. If you don’t have a stapler, buy one. QUizzes WITH UNSTAPLED SHEETS WILL NOT BE GRADED.

4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.

5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.

6. Relax and enjoy...

7. This quiz is due on Monday February 8, in class. NO LATE OR UNSTAPLED QUIZZES WILL BE ACCEPTED.

Pledge: I, ____________________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.
Adapted from Zill & Shanahan, pg. 70, #33.

1. (10 points) A fixed point of a mapping \( w = f(z) \) is a point \( z_0 \) where \( f(z_0) = z_0 \).

   (a) (2 points) Does the linear mapping \( f(z) = az + b \) have a fixed point \( z_0 \)? If so, find \( z_0 \) in terms of the values of \( a \) and \( b \). (NOTE: \( a \) and \( b \) can be any number in the complex plane).

   (b) (2 points) Give an example of a complex linear mapping (i.e. choose values for \( a \) and \( b \)) so that \( f(z) \) has no fixed points.

   (c) (2 points) Give an example of a complex linear mapping (i.e. choose values for \( a \) and \( b \)) so that \( f(z) \) has more than one fixed point. [HINT: There is only one such mapping possible!]

   (d) (4 points) The inverse mapping \( g(w) = cw + d \) is the complex linear mapping such that \( g(f(z)) = z \) and \( f(g(w)) = w \). In other words \( z = g(w) \) “undoes” whatever the mapping \( w = f(z) \) does. Find the values of \( c \) and \( d \) (in terms of the parameters \( a \) and \( b \)) so that \( g(w) = cw + d \) is the inverse mapping of \( f(z) \). Confirm that your choice for \( z = g(w) \) is indeed the inverse of \( w = f(z) \).