MATH312 HW 8

5.1: 6, 7, 8, 11, (27), 20*, 33*

5.2: 2, 7, 10, 21, (22), 29*

6. $\int_{\ln 2}^{\ln 3} e^{-x}\,dx = -e^{-x}\Big|_{\ln 2}^{\ln 3} = -e^{-\ln 3} - (-e^{-\ln 2})$

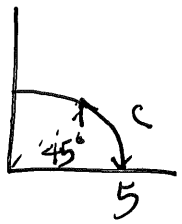
$= e^{-\ln 2} - e^{-\ln 3} = \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$

7. $\int_2^4 x e^{-x^2}\,dx = \frac{e^{-x^2}}{-2}\Big|_2^4 = \frac{e^{-4}}{2} - \frac{e^{-16}}{2}$

8. $\int_1^e \ln x\,dx = x\ln x - x\Big|_1^e = (e\ln e - e) - (1\cdot\ln 1 - 1)$

$= 0 - -1 = \boxed{1}$

11. $\int_C G(x,y)\,dx \qquad G(x,y) = 2xy \qquad x = 5\cos\theta,\ y = 5\sin\theta,\ 0 \le \theta < \pi/4$



$\int_0^{\pi/4} 2(5\cos\theta)(5\sin\theta)\cdot(-5\sin\theta)\,d\theta$

$= -250\int_0^{\pi/4} \cos\theta\cdot\sin^2\theta\,d\theta = -250\int_0^{\sqrt{2}/2} u^2\,du = -250\cdot\frac{u^3}{3}\Big|_0^{\sqrt{2}/2}$

$= -\frac{250}{3}\cdot\frac{1}{2\sqrt{2}} = \boxed{\frac{-125}{3\sqrt{2}}}$

$\int_C G(x,y)\,dy = \int_0^{\pi/4} 2\cdot(5\cos\theta)(5\sin\theta)\,5\cos\theta\,d\theta = 250\int_0^{\pi/4}\sin\theta\cos^2\theta\,d\theta$

$= -250\left(\frac{\cos^3\theta}{3}\right)\Big|_0^{\pi/4} = -\frac{250}{3}\left(\frac{1}{2\sqrt{2}} - 1\right) = \frac{125}{3}\left(\frac{4-\sqrt{2}}{2}\right)$

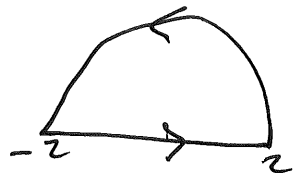
$ds = 5\,d\theta$

$\int_C G(x,y)\,ds = \int_0^{\pi/4} 2\cdot(5\cos\theta)\cdot(5\sin\theta)\,5\,d\theta = 250\int_0^{\pi/4}\frac{\sin 2\theta}{2}\,d\theta$

$= 250\left(-\frac{\cos(2\theta)}{4}\right)\Big|_0^{\pi/4} = -\frac{250}{4}\left(-\cos\frac{\pi}{2} - -1\right) = +\frac{250}{4}$

MATH 32 / HW 8

(27)



$x^2+y^2=4$

$x = 2\cos\theta$
$y = 2\sin\theta$

$$\int_C (x^2+y^2)\,dx - 2xy\,dy = \int_{-2}^{2} x^2\,dx + \int_0^{\pi} 4(2\sin\theta)\,d\theta$$

$$= \frac{x^3}{3}\Big|_{-2}^{2} + 8\cos\theta\Big|_0^{\pi} - 8\int_0^{\pi}\cos^2\theta\sin\theta\,d\theta - \int_0^{\pi} 2\cdot 2\cos\theta\cdot 2\sin\theta\, 2\cos\theta\,d\theta$$

$$= \frac{8}{3} - -\frac{8}{3} - 8 - 8 + 16\left(\frac{\cos^3\theta}{3}\right)\Big|_0^{\pi}$$

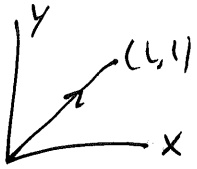
$$= \frac{16}{3} + 16\left(-\frac{2}{3}\right)$$

$$= -\frac{48}{3} - \frac{16}{3} = \boxed{-\frac{64}{3}}$$

20*

$$\int y\,dx + x\,dy \qquad \text{where } y = x \text{ from } (0,0) \text{ to } (1,1)$$

$dy = dx$



$$\int_0^1 x\,dx + \int_0^1 y\,dy = \frac{1}{2} + \frac{1}{2} = 1$$

33* $\int_C y^2\,dx + xy\,dy$

$C: x = 2t+1 \quad dx = 2dt$
$y = 4t+2 \quad dy = 4dt$

$$\int_0^1 (4t+2)^2\,2\,dt + \int_0^1 2(2t+1)^2\cdot 4\,dt = 8\int_0^1 (2t+1)^2\,dt + 8\int_0^1 (2t+1)^2\,dt$$

$$= 16\int_0^1 (2t+1)^2\,dt = \frac{16}{2}\frac{(2t+1)^3}{3}\Big|_0^1$$

$$= \frac{8}{3}(3^3-1) = \frac{208}{3}$$

$C: x = t^2, \; y = 2t^2 \quad 0 \le t \le \sqrt{3}$

$dx = 2t\,dt$
$dy = 4t\,dt$

$$\int_0^{\sqrt{3}} (2t^2)^2\cdot 2t\,dt + \int_{\sqrt{2}}^{\sqrt{3}} 2t^4\cdot 4t\,dt = \int_0^{\sqrt{3}} 8t^5 + 8t^5\,dt = 16\cdot\frac{t^6}{6}\Big|_0^{\sqrt{3}} = \frac{8}{3}(3^3-1)$$

$$= \frac{208}{3}$$

MATH312
HW8

30* cont'd

$x = \ln t,\ y = 2\ln t,\ 1 \le t \le e^3$

$dx = \frac{dt}{t}$

$dy = 2\frac{dt}{t}$

$$\int_C y^2\,dx + xy\,dy$$

$$= \int_e^{e^3} (2\ln t)^2 \frac{dt}{t} + \int_e^{e^3} (2\ln t)^2 \frac{dt}{t} = 8\int_e^{e^3} (\ln t)^2 \frac{1}{t}\,dt$$

$u = \ln t$, $du = \frac{1}{t}dt$, $t = e,\ u = 1$, $t = e^3,\ u = 3$

$$= 8\int_1^3 u^2\,du = 8\,\frac{u^3}{3}\Big|_1^3 = \frac{8}{3}(3^3 - 1)$$

$$= \frac{8 \cdot 26}{3} = \frac{208}{3}$$