MATH 312
HW6

4.1.1 3, 4, 7, 8, 17*
4.1.2 23, 31, 34, 42, 44*
4.2 4, 9, 10, 17, 13*
4.3 2, 9, 37, 53*

4.1.1.

3. $f(z) = e^{iz} - e^{-iz}$

$f'(z) = ie^{iz} + ie^{-iz}$

4. $f(z) = ie^{1/z}$

$f'(z) = ie^{1/z} \cdot \left(-\frac{1}{z^2}\right)$

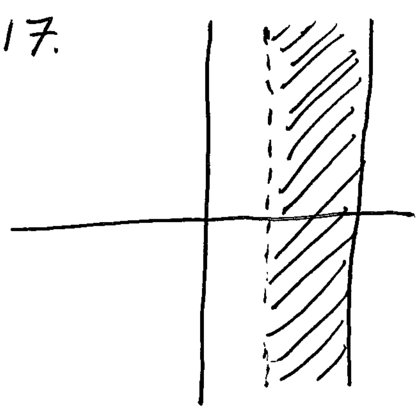
7. $\arg(e^{i(z+\bar{z})}) = \arg(e^{i2x}) = 2x$

8. $\overline{ie^z+1} = \overline{ie^{x+iy}+1} = -ie^{x-iy}+1$

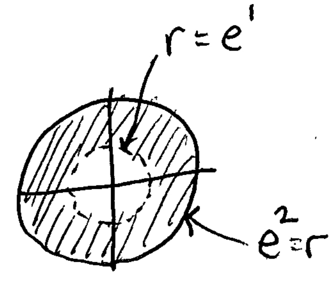
$= -ie^x(\cos y - i\sin y) + 1$

$= e^x \sin y + 1 - ie^x \cos y$

17.



$w = e^z$



$z(t) = \alpha + it$
$-\infty < t < \infty$

$w = e^{\alpha+it} = e^\alpha \cdot e^{it}$
$-\infty < t < \infty$

$S = \{z \in \mathbb{C} : 1 < Re(z) \leq 2\}$

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4.1.2 23, 31, 34, 42, 44*

23. $\ln(-2+2i) = \ln|-2+2i| + i\arg(-2+2i)$

$$= \ln\sqrt{8} + i\frac{3\pi}{4} + 2n\pi i, \ n \in \mathbb{Z}$$

31. $\operatorname{Ln}\left[(1+i\sqrt{3})^5\right] = 5\operatorname{Ln}(1+i\sqrt{3})$

$$= 5\{|1+i\sqrt{3}| + i\operatorname{Arg}(1+i\sqrt{3})\}$$

$$= 5\{2 + i\tfrac{\pi}{3}\} = 10 + \frac{5\pi}{3}i$$

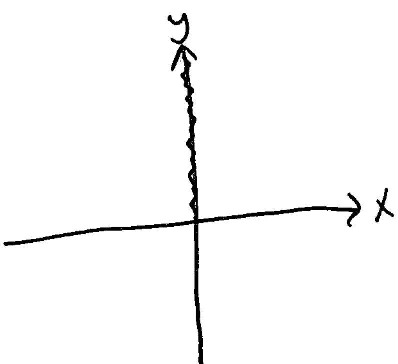
34. $e^{1/z} = -1$

$$\frac{1}{z} = \ln(-1) = \ln|-1| + i\arg(-1)$$

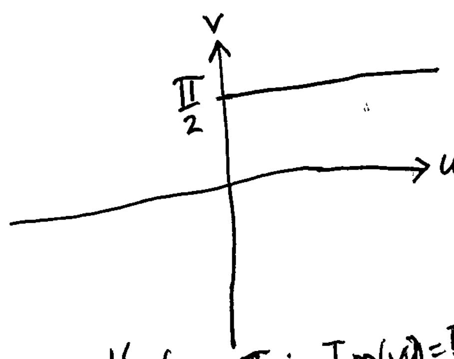
$$\frac{1}{z} = i\pi + 2k\pi i$$

$$z = \frac{1}{i\pi(1+2k)} = \frac{-i}{\pi(1+2k)}, \ k \in \mathbb{Z}$$

42.



$w = \operatorname{Ln} z$



$S = \{z \in \mathbb{C} : \operatorname{Re}(z) = 0 \wedge \operatorname{Im}(z) > 0\}$

$z(t) = it, \ t > 0$

$S' = \{w \in \mathbb{C} : \operatorname{Im}(w) = \frac{\pi}{2} \wedge \operatorname{Re}(w) > 0\}$

$w = \operatorname{Ln}(it) = |it| + i\operatorname{Arg}(it)$

$= |t| + \frac{\pi}{2}i$

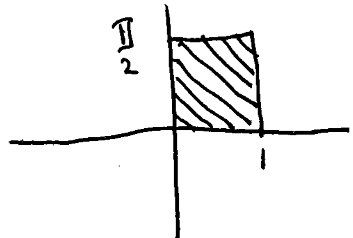
44*

$w = \operatorname{Ln} z$

$z(t) = \rho e^{it}, \ 0 < t < \frac{\pi}{2}$

$w = \operatorname{Ln}(\rho e^{it})$

$= \operatorname{Ln}\rho + it$



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4.2: 4, 9, 10, 17, 13*

4. $(1+\sqrt{3}i)^i = \exp[i \ln(1+i\sqrt{3})]$

$= \exp[i\{\ln|1+i\sqrt{3}| + i\arg(1+i\sqrt{3})\}]$

$= \exp[i\{\ln 2 + i\frac{\pi}{3} + 2n\pi i\}]$

$= \exp[-\frac{\pi}{3}(1+6n) + i\ln 2]$

9. $2^{4i} = \exp[\mathrm{Ln}(2^{4i})] = \exp[4i\,\mathrm{Ln}\,2]$

$= \exp[4i\ln 2] = \cancel{e^4 \cdot e^i \cdot e^{\ln 2} = 2e}$

10 $i^{i/\pi} = \exp[\mathrm{Ln}(i^{i/\pi})] = \exp[\frac{i}{\pi}\mathrm{Ln}\,i]$

$= \exp \frac{i}{\pi}[\log_e|i| + i\,\mathrm{Arg}\,i)$

$= \exp \frac{i}{\pi}[i\frac{\pi}{2}] = \exp(-\frac{1}{2}) = \frac{1}{\sqrt{e}}$

17. $f(z) = z^{1+i} \quad f'(z) = (1+i)z^i$

$f'(1+\sqrt{3}i) = (1+i)(1+i\sqrt{3})^i = \cancel{\ldots}$

$= (1+i)\exp[i\ln(1+i\sqrt{3})]$

$= (1+i)\exp[i\log_e 2 + \frac{\pi}{3}i + 2n\pi i]$

$= (1+i)(\exp[-\frac{\pi}{3}(1+6n) + i\ln 2]$

$= \sqrt{2}e^{i\frac{\pi}{4}} \cdot e^{-\frac{\pi}{3}(1+6n) + i\ln 2} = \sqrt{2}\,e^{-\frac{\pi}{3}(1+6n) + i(\frac{\pi}{4} + \ln 2)}$

13*

$\frac{z^{\alpha_1}}{z^{\alpha_2}} = \exp[\alpha_1 \ln z]\exp[-\alpha_2 \ln z]$

$= \exp[\alpha_1 \ln z - \alpha_2 \ln z] = \exp[(\alpha_1 - \alpha_2)\ln z]$

$= z^{\alpha_1 - \alpha_2}$

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Sec 4.3 ②, 9, 37, 53*

② $\cos(-3i) = \dfrac{e^{i(-3i)} + e^{-i(-3i)}}{2}$

$= \dfrac{e^{3} + e^{-3}}{2} = \cosh(3)$

9. $\sin z = i$

$\Rightarrow \dfrac{e^{iz} - e^{-iz}}{2i} = i \Rightarrow e^{iz} - e^{-iz} = -2$

$w - \frac{1}{w} = -2$ (let $w = e^{iz}$)

$w^2 - 1 = -2w$

$w^2 + 2w - 1 = 0$

$w = \dfrac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2}$

$w = \dfrac{-2 \pm \sqrt{8}}{2} = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$w = -1 \pm \sqrt{2}$

$e^{iz} = -1 \pm \sqrt{2}$

$iz = \ln(-1 \pm \sqrt{2})$

$z = -i \ln(-1 \pm \sqrt{2})$

$z = -i[\ln(-1+\sqrt{2}) + 2n\pi i]$

or

$-i[\ln(1+\sqrt{2}) + 2n\pi i]$

$z = 2n\pi - \ln(\sqrt{2}-1)$

or

$2n\pi - \ln(\sqrt{2}+1)$

37 $e^{iz} = \cos z + i \sin z = \dfrac{e^{iz} + e^{-iz}}{2} + i\left(\dfrac{e^{iz} - e^{-iz}}{2i}\right)$

$= \dfrac{e^{iz}}{2} + \dfrac{e^{-iz}}{2} + \dfrac{e^{iz}}{2} - \dfrac{e^{-iz}}{2}$

$= e^{iz}$ ✓

53* $\sin(z+\pi) = -\sin z$

$\sin(z+\pi) = \dfrac{e^{i(z+\pi)} - e^{-i(z+\pi)}}{2i}$

$= \dfrac{e^{i\pi}e^{iz} - e^{-iz} \cdot e^{-i\pi}}{2i}$

$= \dfrac{-e^{iz} + e^{-iz}}{2i} = -\left(\dfrac{e^{iz} - e^{-iz}}{2i}\right) = -\sin z$

$\cos(z+\pi) = -\cos z$

$\dfrac{e^{i(z+\pi)} + e^{-i(z+\pi)}}{2}$

$= \dfrac{e^{i\pi}e^{iz} + e^{-iz}e^{-\pi i}}{2} = -\left(\dfrac{e^{iz} + e^{-iz}}{2}\right)$

$= -\cos z$