L1

HW Set 4

Chap 2 Review 1-10 (1) (2) (3) (4) (6) (7)

3.1.1 (2), 14, (17), (20*)

3.1.2 (28), 34, (37), (50*)

(1.) "If $f(z)$ is a complex function, then $f(x+0i)$ must be a real number." $\boxed{\text{FALSE}}$

Clearly, $f(z) = iz$ is an example of a function where $f(x+0i) = ix$ which is NOT a real number

(2.) "$\arg(z)$ is a complex function." $\boxed{\text{FALSE.}}$

$\arg(z)$ outputs multiple values for each input so it is NOT a function.

(3.) "The domain of the function $f(z) = \frac{1}{z^2+i}$ is all complex numbers." $\boxed{\text{FALSE}}$

The values $z^2+i=0$ are NOT in the domain of $f(z)$. $z = \sqrt{-i} = e^{-\frac{\pi i}{4}}, e^{\frac{3\pi i}{4}}$

(4.) "The domain of the function $f(z) = e^{z^2-(1+i)z+2}$ is all complex numbers." $\boxed{\text{TRUE}}$

The exponential function takes all values as its input.

5. "If $f(z)$ is a complex function with $u(x,y)=0$, then the range of f lies in the imaginary axis." $\boxed{\text{TRUE}}$

$f(z) = u(x,y) + iv(x,y)$ so if $u(x,y)=0$ then $f = iv(x,y)$

(6.) "The entire complex plane is mapped onto the real axis $v=0$ by $w = z+\bar{z}$." $\boxed{\text{TRUE}}$

$w = (x+iy)+(x-iy) = 2x$ where $x \in \mathbb{R}$ so $w \in \mathbb{R}$

7. "The entire complex plane is mapped onto the unit circle $|w|=1$ by $w = \frac{z}{|z|}$."

$\boxed{\text{FALSE}}$ $\boxed{\cancel{\text{TRUE}}}$ $|w| = \left|\frac{z}{|z|}\right| = \frac{|z|}{||z||} = \frac{|z|}{|z|} = 1$ ✓

$0 \in \mathbb{Z}$ not in set so, entire complex plane not mapped

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8. "The range of the function $f(z) = \text{Arg}(z)$ is all real numbers." $\boxed{\text{FALSE}}$

The range of $\text{Arg}(z)$ is $\{x \in \mathbb{R} : -\pi < x \leq \pi\}$.

9. "The image of the circle $|z - z_0| = \rho$ under a linear mapping is a circle with a possibly different center, but the same radius." $\boxed{\text{FALSE}}$

$w = 2z$ is a linear mapping and it changes $|z - z_0| = \rho$ to be $|w - 2z_0| = 2\rho$. center moved and radius increased.

$z(t) = z_0 + \rho e^{it}, \ 0 \leq t \leq 2\pi$

$w = 2z = 2z_0 + 2\rho e^{it}, \ 0 \leq t \leq 2\pi$

10. "The linear mapping $w = (1 - \sqrt{3}i)z + 2$ acts by rotating through an angle of $\pi/3$ radians clockwise about the origin, magnifying by a factor of 2, then translating by 2." $\boxed{\text{TRUE}}$

$$R(z) = e^{-\frac{\pi i}{3}} z$$
$$S(z) = 2z$$
$$T(z) = 2$$
$$w(z) = T(S(R(z))) = T(S(e^{-\frac{\pi i}{3}} z))$$
$$= T(2e^{-\frac{\pi i}{3}} z)$$
$$= 2e^{-\frac{\pi i}{3}} z + 2$$
$$= 2\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) z + 2$$
$$= (1 - i\sqrt{3}) z + 2 \ \square$$

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3.1.1. 2, 11, 17, 20*.

② $\lim_{z\to 1+i} \frac{z-\bar{z}}{z+\bar{z}} = \lim_{(x,y)\to(1,1)} u(x,y)+iv(x,y) = i$

$\frac{z-\bar{z}}{z+\bar{z}} = \frac{(x+iy)-(x-iy)}{(x+iy)+(x-iy)} = \frac{2iy}{2x} = \frac{iy}{x} \Rightarrow \begin{matrix} u=0 \\ v=\frac{y}{x} \end{matrix}$

⑪ $\lim_{z\to e^{\frac{i\pi}{4}}} \left(z+\frac{1}{z}\right) = e^{\frac{i\pi}{4}} + \frac{1}{e^{\frac{i\pi}{4}}} = e^{\frac{i\pi}{4}} + e^{\frac{-i\pi}{4}} = 2\cos\left(\frac{\pi}{4}\right)$
$= \sqrt{2}$

⑰ $\lim_{z\to 0} \frac{Re(z)}{Im(z)} = \lim_{(x,y)\to(0,0)} \frac{x}{y}$

(a) If $z\to 0$ along $y=x$, $\lim_{z\to 0} \frac{Re(z)}{Im(z)} = \lim_{(x,y)\to(0,0)} 1 = 1$

(b) If $z\to 0$ along imaginary axis ($x=0$)
$\lim_{z\to 0} \frac{Re(z)}{Im(z)} = \lim_{(x,y)\to(0,0)} 0 = 0$

(c) The limit does not exist!

⑳* $\lim_{z\to 0} \left(\frac{2y^2}{x^2} - \frac{x^2-y^2}{y^2} i\right)$

(a) suppose $y=x$
$\lim_{(x,y)\to(0,0)} 2-0i = 2$

(b) suppose $y=-x$
$\lim_{(x,y)\to(0,0)} 2-0i = 2$

(c) Answers imply that limit exists since we get same answer along both paths

(d) Suppose $y=2x$
$\lim_{(x,y)\to(0,0)} \frac{2(2x)^2}{x^2} - \frac{x^2-(2x)^2}{(2x)^2} i$
$= \lim_{(x,y)\to(0,0)} 8 - \frac{3}{4}i = 8+\frac{3i}{4}$

(e) limit does not exist!

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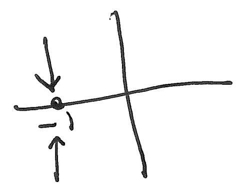
3.1.2 28, 31, 37, 50*

28. $f(z) = z^3 - \frac{1}{z}$ $\quad z_0 = 3i$

$$\lim_{z \to 3i} f(z) = (3i)^3 - \frac{1}{3i} = -27i - \frac{1}{3}(-i) = -\frac{80i}{3}$$

31. $\lim_{z \to 1} f(z)$ $\qquad f(z) = \begin{cases} \frac{z^3 - 1}{z - 1}, & |z| \neq 1 \\ 3, & |z| = 1 \end{cases}$

$$\lim_{z \to 1} \frac{(z-1)(z^2+1)}{(z-1)} = \lim_{z \to 1} z^2 + 1 = 2$$



37. $\lim_{z \to -1} \operatorname{Arg} z = \text{DNE}$

$\lim_{z \to -1} \operatorname{Arg} z = \pi$ when $z \to -1$ from above

$\lim_{z \to -1} \operatorname{Arg} z = -\pi$ when $z \to -1$ from below

50* $\lim_{z \to z_0} \bar{z} = \bar{z}_0$ if for every $\epsilon > 0$, there is a $\delta > 0$ such that $|\bar{z} - \bar{z}_0| < \epsilon$ whenever $0 < |z - z_0| < \delta$.

By properties of complex modulus and conjugation, $|z - z_0| = |\overline{z - z_0}| = |\bar{z} - \bar{z}_0|$. Therefore, if $0 < |z - z_0| < \delta$ and $\delta = \epsilon$, then $|\bar{z} - \bar{z}_0| < \epsilon$.