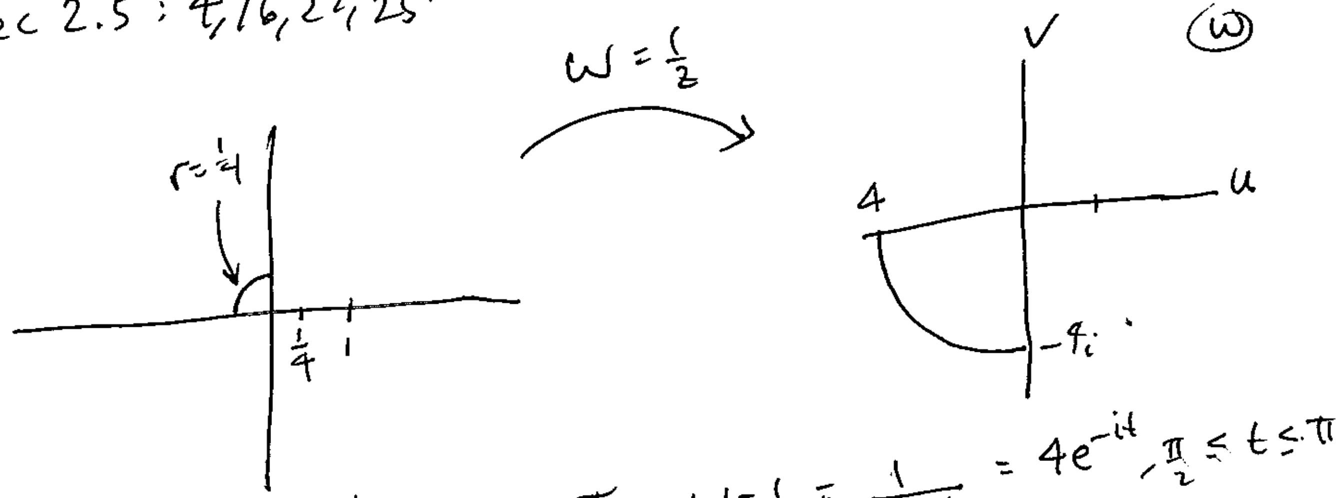
# HW Set 3



Sec 2.5 : 4, 16, 22, 25*

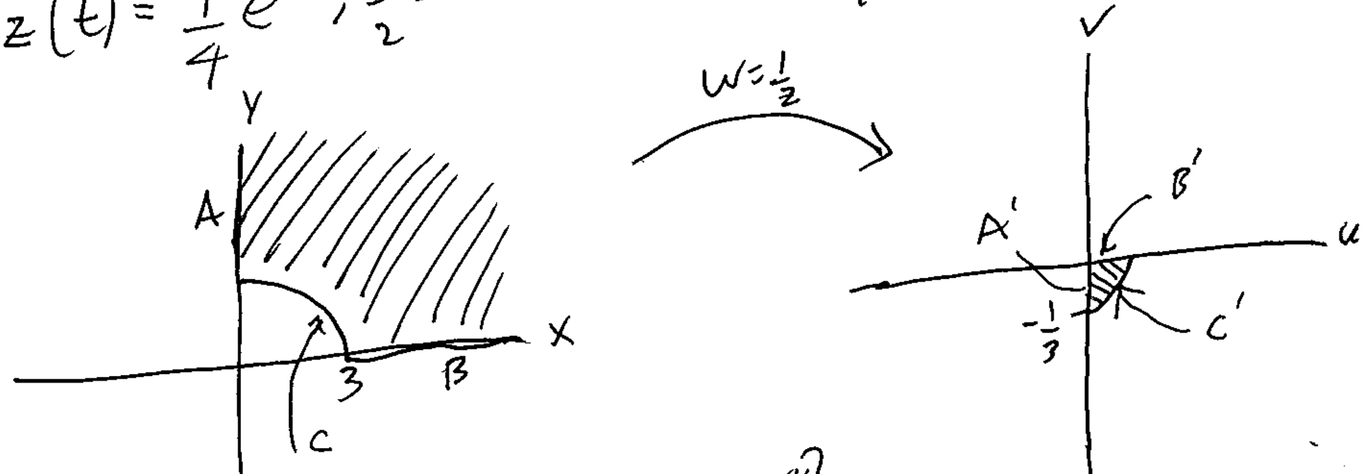
4.




$w = \frac{1}{z} = \frac{1}{\frac{1}{4}e^{it}} = 4e^{-it}, \frac{\pi}{2} \leq t \leq \pi$

$z(t) = \frac{1}{4}e^{it}, \frac{\pi}{2} \leq t \leq \pi$

16.




$S = \{ z \in \mathbb{C} \mid |z| > 3 \cap 0 < \text{Arg} z < \frac{\pi}{2} \}$

$S' = \{ w \in \mathbb{C} : |w| < \frac{1}{3}$

A: $z_1 = it, \quad t \geq 3$

B: $z_2 = t, \quad t > 3$

C: $z_3 = 3e^{it}, \ 0 < t < \frac{\pi}{2}$

$w_1 = \frac{1}{it}, \ t > 3$

$= \frac{-it}{t}, \ t > 3$

$w_2 = \frac{1}{t}, \quad t > 3$

$w_3 = \frac{1}{3}e^{-it}, \ 0 < t < \frac{\pi}{2}$

# HW Set 3

22. $h(z) = \frac{3i}{z^2} + 1 + i$

$f_1(z) = 3iz + 1 + i$

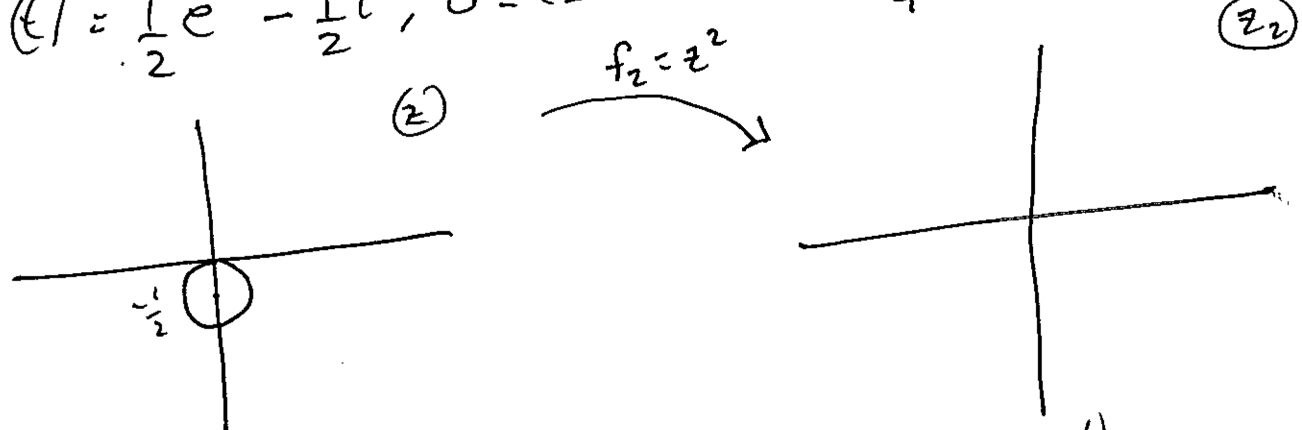
$f_2(z) = z^2$

$f_3(z) = \frac{1}{z}$

$h = f_1 \circ f_3 \circ f_2(z) = f_1(f_3(f_2(z)))$

(b) $|z + \frac{1}{2}i| = \frac{1}{2}$ is a circle of radius $\frac{1}{2}$ centered at $(0, -\frac{1}{2})$

$z(t) = \frac{1}{2}e^{it} - \frac{1}{2}i, \; 0 \le t \le 2\pi$ $\qquad z_2 = \frac{1}{4}e^{2it} - \frac{1}{4}ie^{it} - \frac{1}{4}, \; 0 \le t \le 2\pi$

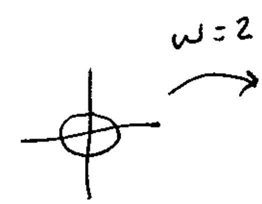



$z_2 = (\frac{1}{4}\cos 2t + \frac{1}{4}\sin t) - \frac{1}{4} + i(\frac{1}{4}\sin 2t - \frac{1}{4}\cos t)$

$= \frac{u}{4} + i\frac{v}{4}$

$(u+1)^2 = \cos^2 2t + \sin^2 t + 2\cos 2t \sin t$

$v^2 = \sin^2 2t + \cos^2 t - 2\sin 2t \cos t$




$(u-1)^2 + (v-1)^2 = 1$

$u^2 + v^2 = 1$

$3v = \frac{9u^2}{4} - 1$

$(2u)^2 + (2v)^2 = 1$

$u^2 + v^2 = \frac{1}{4}$

$v = \frac{u^2 - 36}{12}$

$v = \frac{3u^2}{4} - 1$

$\left(\frac{v}{3}\right) = \left(\frac{u}{3}\right)^2 - 1$

$v = \frac{3u^2}{9} - 3$

$v = \frac{1u^2}{3} - 3$

# HW Set 3

22

$$h(z) = \frac{3i}{z^2} + 1 + i$$

$$f_3(z) = 3iz + 1 + i$$

$$f_2(z) = z^2$$

$$f_1(z) = \frac{1}{z}$$

$$h(z) = f_3 \circ f_2 \circ f_1(z) = f_3 \circ f_2\left(\frac{1}{z}\right)$$

$$= f_3\left(\frac{1}{z^2}\right)$$

$$= 3i\frac{1}{z^2} + 1 + i$$

(b)

$\left|z + \frac{1}{2}i\right| = \frac{1}{2}$ is a circle of radius $\frac{1}{2}$ centered at $(0, -\frac{1}{2})$

p. 93 The circle $\left|z + \frac{i}{2}\right| = \frac{1}{2}$ is mapped onto the vertical line $y = ~~1~~$, i.e. $\text{Im}(z) = \frac{i}{1}$

p. 74 The vertical line $y = k$ is mapped onto parabola $u = \frac{v^2}{4k^2} - k^2$

so, if $k = 1$, ~~$u = \frac{v^2}{4}$~~ $u = \frac{v^2}{4} - 1$

This parabola is then rotated 90° counterclockwise and shifted 1 unit to the right and up.

$v = \frac{u^2}{4} - 1$ is rotated parabola

~~$v = 1 + \frac{(u-1)^2}{4}$~~ $- 1$ is parabola shifted 1 unit left and right

$= (u-1)^2$ ~~4~~

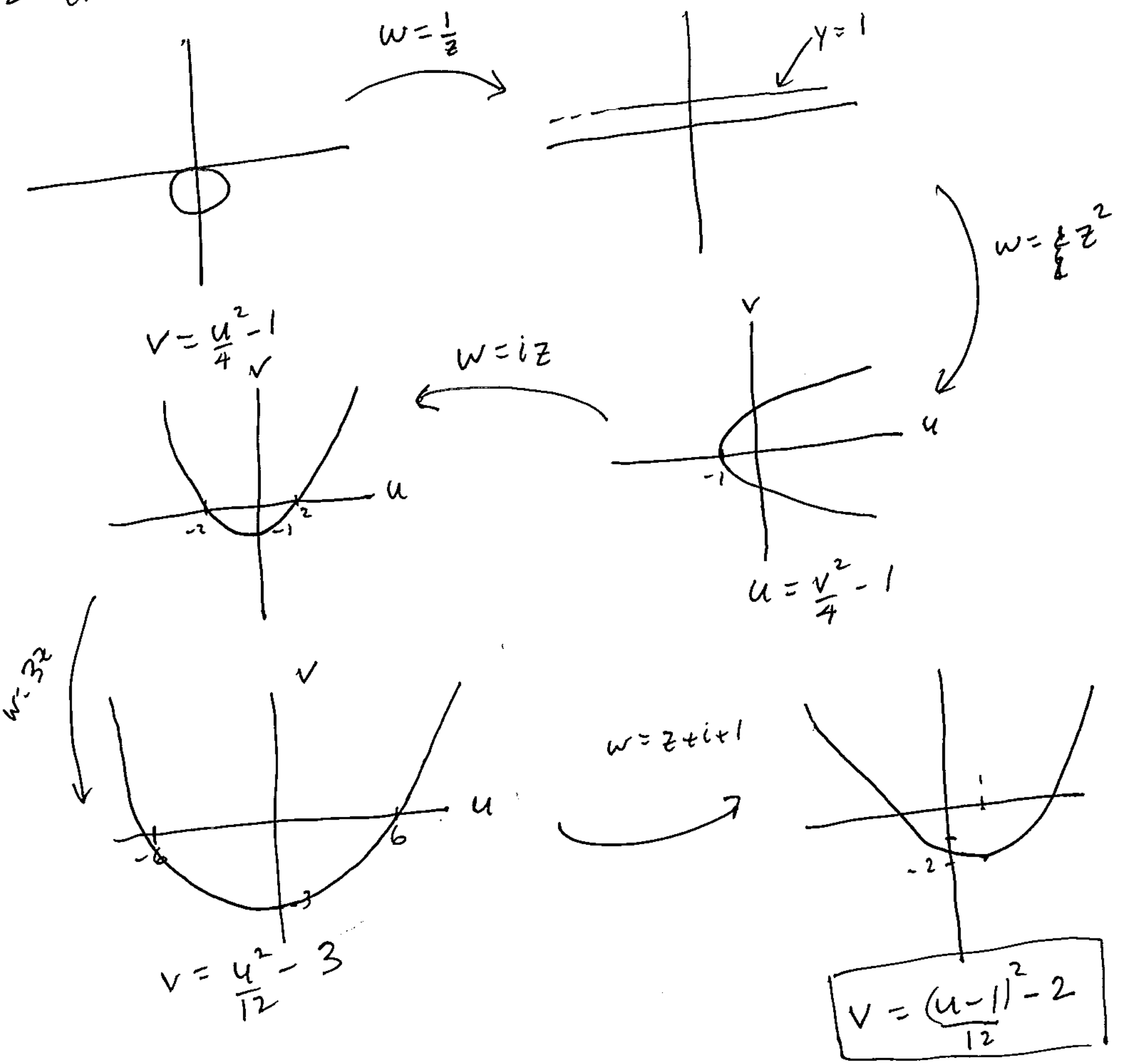
scaled parabola

$\frac{v}{3} = \left(\frac{u}{3}\right)^2 \frac{1}{4} - 1 \Rightarrow v = \frac{u^2}{12} - 3$ ~~4~~ translated parabola: $(v - 1) = \frac{(u-1)^2}{12} - 3$

# HW Set 3 2.5

15

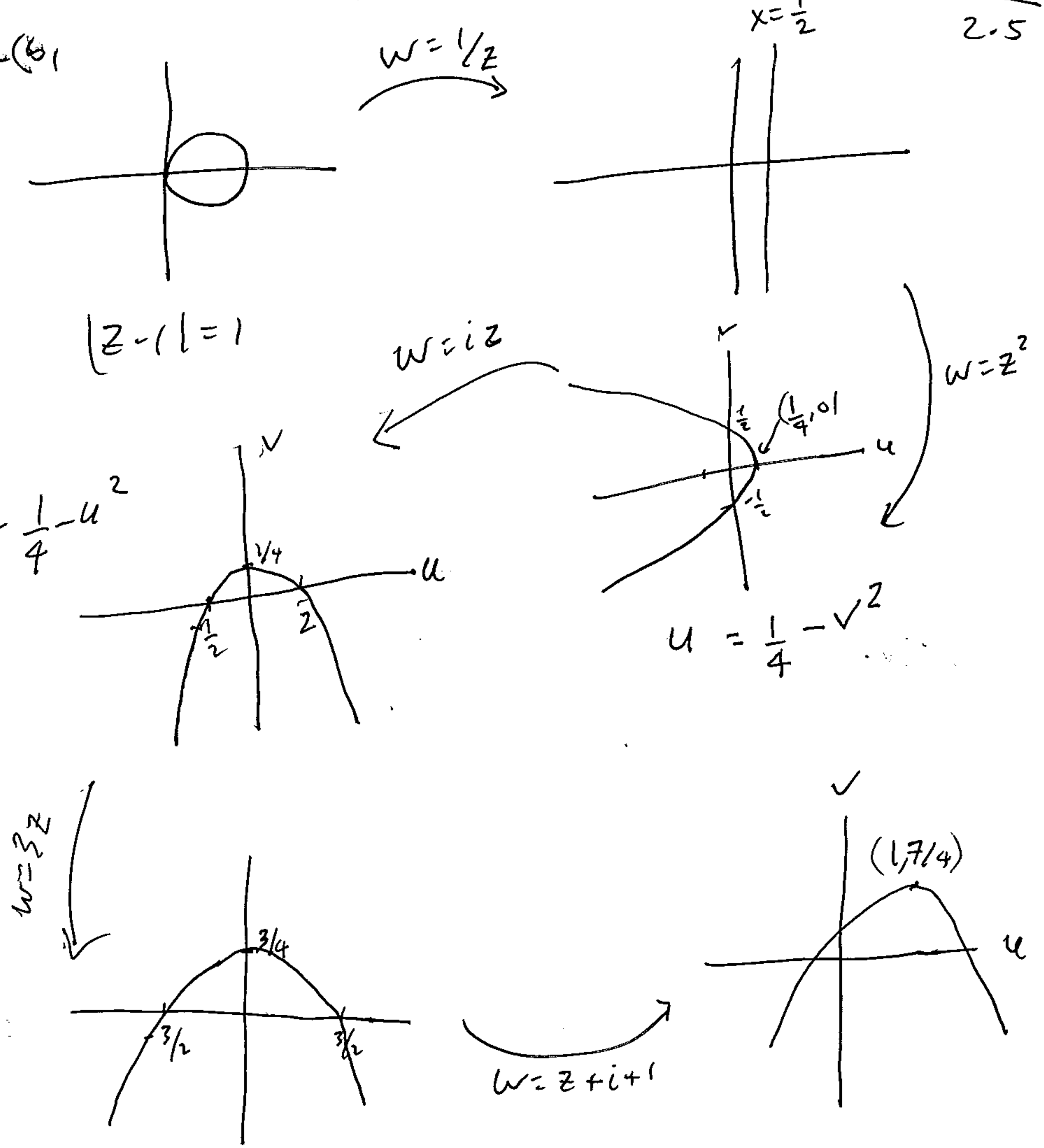
22 (a)



# HW Set



22(b)




$$\left(\frac{v}{3}\right) = \frac{1}{4} - \left(\frac{u}{3}\right)^2$$

$$v = \frac{3}{4} - \frac{u^2}{3}$$

$$v - 1 = \frac{3}{4} - \frac{(u-1)^2}{3}$$

$$v = \frac{7}{4} - \frac{(u-1)^2}{3}$$

$|z-1| = 1$ under $f(z) = 3i\frac{1}{z^2} + i + 1$ is the parabola $v = \frac{7}{4} - \frac{(u-1)^2}{3}$

# MATH 312 HW Set 3

/17

2.5

25*

$$A(x^2+y^2)+Bx+Cy+D=0$$

(a) $A=0 \Rightarrow Bx+Cy+D=0$

$$y = \frac{D}{B}-\frac{B}{A}x$$ which is a line of slope $-B/A$ with intercept $D/B$

(b) $A \neq 0$ $\Delta = B^2+C^2-4AD$

$$Ax^2+Bx+Ay^2+Cy+D=0$$

$$A\left(x^2+\frac{B}{A}x\right)+A\left(y^2+\frac{C}{A}\right)+D$$

$$A\left(x+\frac{B}{2A}\right)^2-\frac{B^2}{4A}+A\left(y+\frac{C}{2A}\right)^2-\frac{C^2}{4A}+D=0$$

$$A\left(x+\frac{B}{2A}\right)^2+A\left(y+\frac{C}{2A}\right)^2=\frac{B^2}{4A}+\frac{C^2}{4A}-D$$

$$=\frac{B^2+C^2-4AD}{4A}$$

$$\left(x+\frac{B}{2A}\right)^2+\left(y+\frac{C}{2A}\right)^2=\frac{B^2+C^2-4AD}{4A^2}$$

$$=\frac{\Delta}{4A^2}$$

$$\left(x+\frac{B}{2A}\right)^2+\left(y+\frac{C}{2A}\right)^2=\left(\frac{\sqrt{\Delta}}{2A}\right)^2$$

This is a circle of radius $\frac{\sqrt{\Delta}}{2A}$ centered at $\left(-\frac{B}{2A},-\frac{C}{2A}\right)$