Complex Analysis

Math 214 Spring 2014 | Fowler 307 MWF 3:00pm - 3:55pm

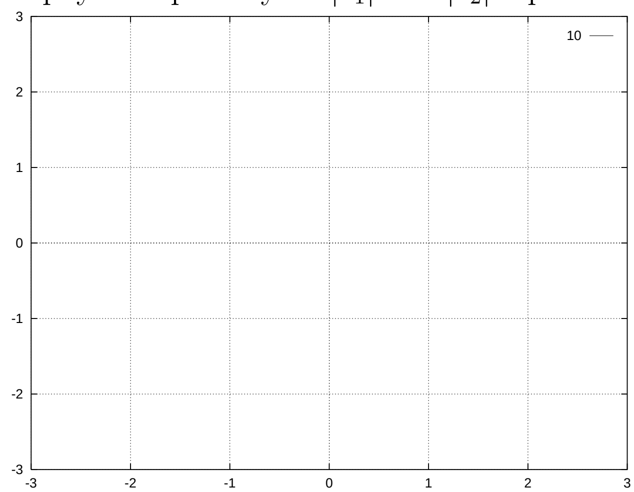
 | http://faculty.oxy.edu/ron/math/312/14/

Class 2: **Friday January 24**

TITLE Graphical Representations of Complex Numbers and Inequalities
READING Zill & Shanahan, Section 1.2
HOMEWORK Zill & Shanahan, Section 1.2 # 2, 5, 20, 29, 33, **Extra Credit: 38**

SUMMARY
We shall be introduced to the graphical (geometrical) representation of complex numbers, particularly the Argand diagram.

Consider two complex numbers $z_1 = 3 + 0.5i$ and $z_2 = -1 - 2i$
Draw an *Argand diagram* depicting these two complex numbers in the complex plane (on the grid provided). What physical quantity do $|z_1|$ and $|z_2|$ represent in the diagram?



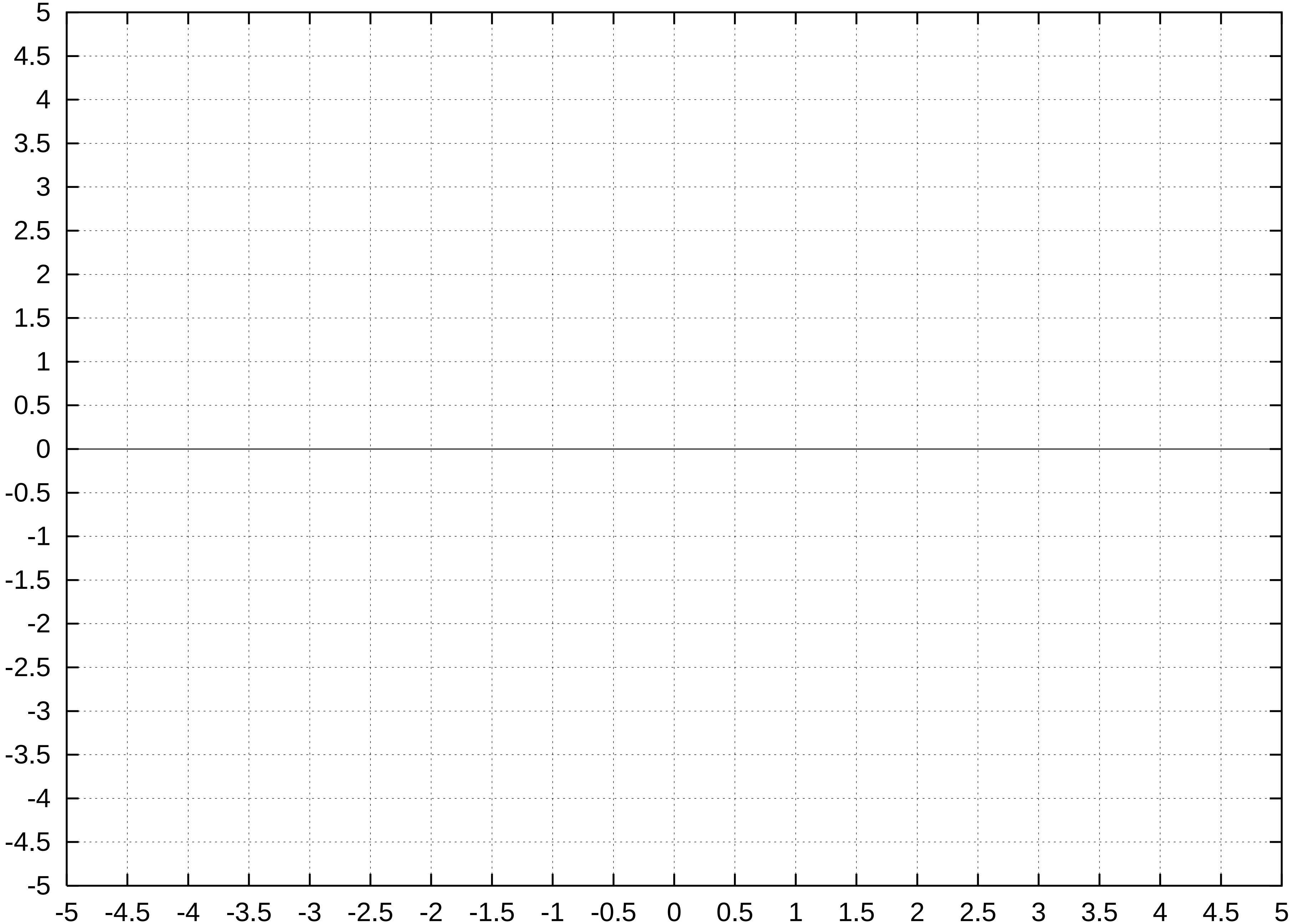
Then draw in vectors that represent the complex numbers $z_1 + z_2$ and $z_1 - z_2$
Indicate what the value of $|z_1 + z_2|$ is. If I had two points at (3,0.5) and (-1,-2) what would the distance between these two points be?

GROUPWORK

Consider the equation $|z - 2 + i| = 2$.
What curve does this equation represent in the complex plane?

Consider the equation $2 = \text{Re}\,(\bar{z} - i)$
What curve does this equation represent in the complex plane? How about $\text{Im}(\bar{z} - i) = 2$?

Sketch the set of points which satisfy each of these equations on the grid provided (on the next page)



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
-3.5
-4
-4.5
-5
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Complex Inequalities
There are a number of interesting inequalities that one can prove using complex variables which have significance in other arenas.
The most famous of these is the *Triangle Inequality*

Triangle Inequality

$$|z_1 + z_2| \leq |z_1| + |z_2| \tag{1}$$

Can you express a geometric or vector interpretation of the Triangle Inequality?

This expression (1) is also true in a more general form known as the *Cauchy-Schwarz Inequality*
Cauchy-Schwarz Inequality

$$\begin{aligned} |z_1 + z_2 + z_3 + \ldots + z_n| &\leq |z_1| + |z_2| + |z_3| + \ldots + |z_n| \\ \left|\sum_{k=1}^{n} z_k\right| &\leq \sum_{k=1}^{n} |z_k| \end{aligned}$$

The Cauchy-Schwarz Inequality can be proved by *mathematical induction.*
PROOF

Some Other Inequalities

For the inequalities below, try to come up with a geometric interpretation and prove that they are true.

$$\text{Re } z \leq |\text{ Re } z| \leq |z| \text{ and } \text{ Im } z \leq |\text{ Im } z| \leq |z|$$

$$|z_1 + z_2| \geq ||z_1| - |z_2|| \tag{2}$$

$$|z_1 - z_2| \leq |z_1| + |z_2| \tag{3}$$

$$|z_1 - z_2| \geq ||z_1| - |z_2|| \tag{4}$$

Exercise

Inequalities (2), (3) and (4) are found on page 12 of Zill & Shanahan. Pick one of them and try and to prove it is true.

EXAMPLE

Show that an upper bound for $\left|\dfrac{-1}{z^4 + 3z^2 + 2}\right|$ when $|z| = 2$ is $\dfrac{1}{6}$.