COLOR GRAPHS OF COMPLEX FUNCTIONS

SIRENA VAN EPP AND ABIGAIL MARTINEZ

After learning about complex functions and simple mappings, we decided to further explore visual representations of complex functions. After experimenting with several types of visuals, we thought it best to split the functions up into four separate graphs and use color to represent the values of the graphs. We chose to split the function representations up into four graphs in order to analyze the real, imaginary, modulus, and arguments of the functions separately. To show different types of imaginary functions, we selected four functions, $z, 2z + z, z^2$, and $\frac{1}{z}$. In particular, we took note of how the compared to one another. In other words, $f(z)$ is our basic function, and we saw how scaling, translations, quadratics, and inverses changed the different parts of z.

We started with a very basic complex function, $f(z) = z$. The real part of the graph is represented by an infinite number of vertical lines, which is reasonable because at every value of x (real number), there will be an infinite amount of y (imaginary) values. The colors we chose are in the ROYGBIV spectrum since most people are aware of the order in which these colors are arranged. The violet and indigo colors are then the smallest values, where as the red and orange are the areas of largest values. In this particular graph, we see that the as x becomes more negative, the spectrum becomes more cool colored and as x increases, the colors range from orange to red.

The next graph $f(z) = \text{Im}(z)$, is similar to the real part of z except that the graph is composed of horizontal lines rather than vertical ones. The lines are horizontal because for every y value there exists an infinite number of x values. We also see a similar pattern with the colors, as y goes from positive to negative, the spectrum goes from warm to cool colors.

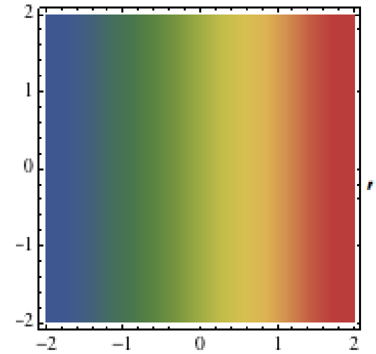

FIGURE 1. Real graph of $f(z) = z$

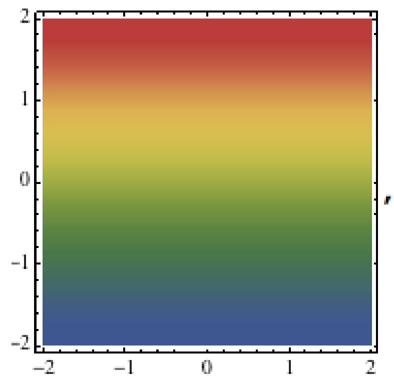

FIGURE 2. Imaginary graph of $f(z) = z$

Our third graph illustrates the modulus of $f(z) = z$. Simply put, modulus is represented by the formula: $|x + iy| = \sqrt{x^2 + y^2}$. Our graph shows that as x and y increase, so do the values of the modulus, which agrees with our intuition that as z increases, so should the modulus.

Our final graph for $f(z) = z$, demonstrates the argument of z. In order to analyze the argument, we start at the real axis and follow a clockwise direction. As the argument reaches 2π, the colors should become warmer (increase). We do see in the graph that as the two axises become more positive, the argument is red.

Next we show what happens when we scale and shift z by a values of 2. Thus our function for the following graphs is $f(z) = 2z + 2$. Looking at the representations of the

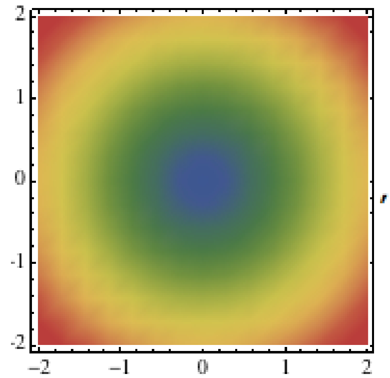

FIGURE 3. Modulus graph of $f(z) = z$

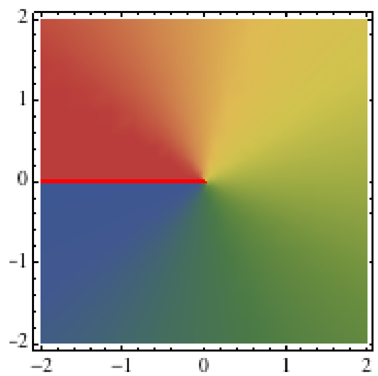

FIGURE 4. Argument graph of $f(z) = z$

real and imaginary parts of the function $f(z) = 2z + 2$, we see no change since the there is still and infinite amount of y values for every x value and vice-versa.

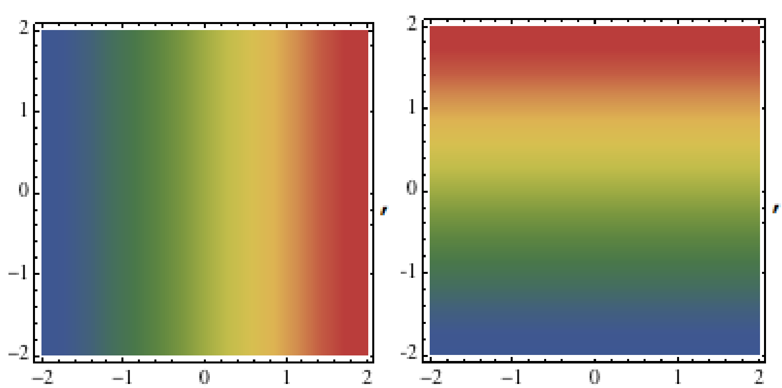

FIGURE 5. Real and imaginary graphs of $f(z) = 2z + 2$

There is a noticeable change however within the modulus of our function. We see that the center is no longer at the origin, rather it has been shifted to the left by two units. The same is for the graph of the argument. This is because we shifted our graph by two units. We also see that the center is larger (the blue are is has grown to twice the size as the z mod graph) since we scaled the graph by a factor of two. In the argument we also see a similar change in that it takes twice as long for the graph to change colors as x and y increase.

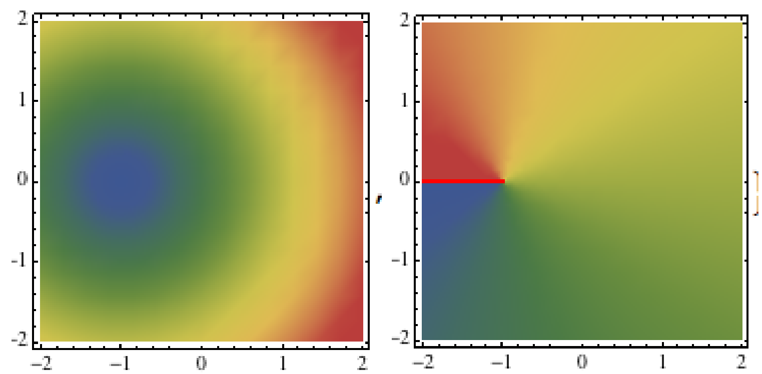

FIGURE 6. Modulus and argument graphs of $f(z) = 2z + 2$

Thirdly, we experimented with the function $f(z) = z^2$. The graph for the real part of $f(z) = z^2$ is significantly different from both of our previous graphs. We can see the the middle of the graph is much smaller the the edges (as x heads towards ∞ or $-\infty$). This makes sense since we know that z^2 will behave much like x^2 does in the real plane. Hence z^2 will grow larger faster as x heads towards ∞ or $-\infty$. The same explanation holds true for the imaginary graph of z^2.

The modulus graph of z^2 is similar to the real and imaginary graphs, in other words the function starts off small then increases rapidly. Again, this graph behaves like the function x^2 in the xyz-plane since they both look like bowls.

The argument graph is distinct from the previous functions' argument graphs in that this is the first time we see warm colors above and below the x-axis. This can be

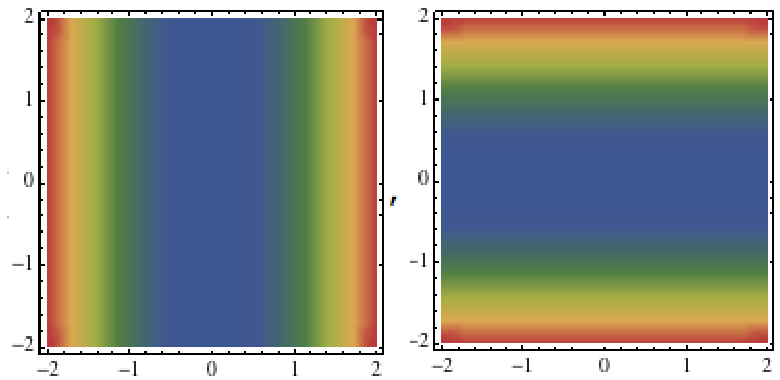

FIGURE 7. Real and imaginary graphs of $f(z) = z^2$

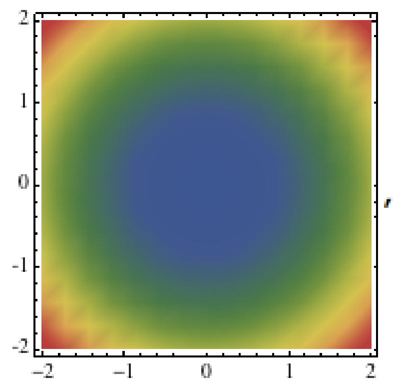

FIGURE 8. Modulus graph of $f(z) = z^2$

explained by the fact that positive and negative can have large arguments due to the properties of a quadratic function.

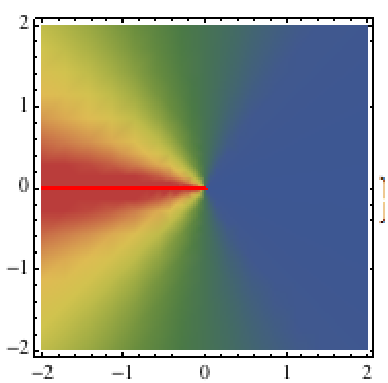

FIGURE 9. Argument graph of $f(z) = z^2$

Finally, we decided to look at the function $f(z) = \frac{1}{z}$. The most significant difference in all of these graphs is that they have sections of white at $z = 0$. This is obvious since $\frac{1}{z}$ is undefined at $z = 0$. It is also important to note that as the z values increase, the function decreases since the limit as z approaches ∞ is 0. Besides the undefined areas, the real, imaginary, and modulus graphs for $f(z) = \frac{1}{z}$ behave more or less like the previous functions. Although the real, imaginary, and modulus graphs behaved as expected, the argument graph does not match our expectations. As you can see, the sections that is undefined is not the line $\text{Im}(z) = 0$. We can only conclude that this is a numerical error due to the program we used. When trying to fix this error we noticed that the undefined area remained between $\text{Im}(z) = \pm 0.2$. Thus we decided that in the overall picture, this error was negligible and our representations are accurate.

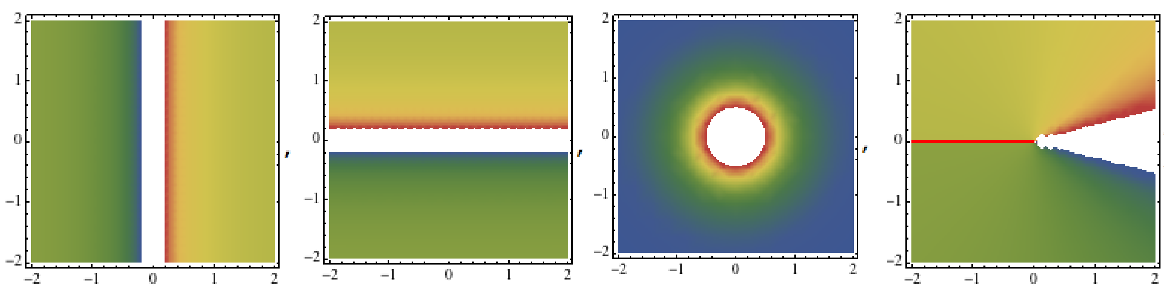

Figure 10. Real, imaginary, modulus, and argument graphs of $f(z) = \frac{1}{z}$

Ultimately, our graphs gave us a deeper understanding of the behavior of complex functions. Through our graphs, we saw the different behaviors of the real, imaginary, modulus, and argument of different functions. In addition, they provided a new fun way to visualize functions in the complex plane.

Note: We created all of the graphs on Wolfram's Mathematica by manipulating the following code:

Table[DensityPlot[f[(x + Iy)], x, -2, 2, y, -2, 2, ColorFunction − > "DarkRainbow", ExclusionsStyle − > Red], f, Re, Im, Abs, Arg]