Complex Analysis

Math 214 Spring 2004 — Fowler 316 MWF 3:30pm - 4:25pm

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Class 31: **Monday April 12**

SUMMARY More Applications of Residues
CURRENT READING Saff & Snider, §6.3 (page 327-328)

In addition to evaluating complicated trigonometric integrals with integrands of the form $F(cos\theta, \sin\theta)$ on the interval $[0, 2\pi]$, Residues can be used to evaluate the sum of an infinite list of numbers, i.e. actually evaluate infinite series exactly.

Using Residues To Evaluate Infinite Series

Using $f(z) = \dfrac{\pi \cot(\pi z)}{p(z)}$ which has a finite number r poles at $z_{p_1}, z_{p_2}, \ldots, z_{p_r}$ where $p(z)$ has **(i)** real coefficients, **(ii)** degree $n \geq 2$ and **(iii)** no integer zeroes then

$$\sum_{k=-\infty}^{\infty} \frac{1}{p(k)} = -\sum_{j=1}^{r} \textbf{Res}\left(\frac{\pi \cot(\pi z)}{p(z)}, z_{p_j}\right)$$

Using $g(z) = \dfrac{\pi \csc(\pi z)}{p(z)}$ where $p(z)$ has the same conditions as before, then

$$\sum_{k=-\infty}^{\infty} (-1)^k \frac{1}{p(k)} = -\sum_{j=1}^{r} \textbf{Res}\left(\frac{\pi \csc(\pi z)}{p(z)}, z_{p_j}\right)$$

EXAMPLE

We'll obtain the result that $\displaystyle\sum_{k=-\infty}^{\infty} \frac{1}{k^2+a^2} = \frac{\pi}{a}\coth(\pi a)$ and use this to show that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$

GROUPWORK

Show that $\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} = \frac{\pi^2}{8}$