Complex Analysis

Math 214 Spring 2004 | Fowler 112 MWF 3:30pm - 4:25pm

 | http://faculty.oxy.edu/ron/math/312/04/

Class 22: **Monday March 22**

SUMMARY Independence of Path
CURRENT READING Saff & Snider, §4.3
HOMEWORK Saff & Snider, Section 4.3 # 1,2,4,5,7

We have been practicing our complex integration skills. Today we will learn how to use **antiderivatives** and the **Cauchy-Goursat Theorem** to evaluate contour integrals more efficiently (simply!)

Recall that the value of $\int_C 2\overline{z}^2\,dz$ **did** depend on the contour when we evaluated it in the previous class.

However the value of $\int_C 2z^2\,dz$ **did not** depend on the contour chosen and in each case the integral is equal to $\frac{-32}{3}$

How is $f(z) = 2z^2$ a different function than $g(z) = 2\overline{z}^2$?

$2z^2$ is a ________________________ while $2\overline{z}^2$ is a ________________________.

Independence of Path

THEOREM: Suppose that the function $f(z)$ is continuous in a domain (open connected set) D and has an antiderivative $F(z)$ throughout D, i.e. $dF/dz = f(z)$ at each point in D. Then for *every* contour Γ lying in D connecting z_1 to z_2 we have the result

$$\int_\Gamma f(z)\,dz = F(z_2) - F(z_1)$$

In otherwords, the value of the integral is **independent of the path** chosen to link z_1 and z_2

So, getting back to the difference between $f(z) = 2z^2$ and $g(z) = 2\overline{z}^2$ we know that $2z^2$ has the property that it is continuous and is equal to the derivative of $2z^3/3$ on the open set $z \in \mathbf{C}$. $g(z)$ has no similar antiderivative. (This is not surprising, since $g(z)$ doesn't have a derivative either.)

Example

$\int_C 2z^2\,dz =$ where C is a contour from 2 to -2

Note that according to the above theorem the function $F(z)$ will be **analytic** and **continuous** on the domain D (since it has a derivative at every point of the open set D).
A corollary of the above theorem is the famous
Cauchy-Goursat Theorem: If $f(z)$ is analytic at all points interior to and on any simple closed contour Γ, then

$$\oint_\Gamma f(z)dz = 0.$$

(Is this result surprising?)
<u>GROUPWORK</u>
1. $\displaystyle\int_C \sin(iz) + 2e^z\, dz,$ where $C$is a contour joining 0 to πi

2. $\displaystyle\oint_C \frac{dz}{z^2-4}\, dz,$ where $C : |z| = 1$

3. $\displaystyle\int_{-i}^{i} \frac{dz}{z^2}$ and $\displaystyle\oint_{C_r} \frac{dz}{z^2}$ where $C_r : |z| = 1$