
Complex Analysis

Math 214 Spring 2004
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Fowler 112 MWF 3:30pm - 4:25pm
<http://faculty.oxy.edu/ron/math/312/04/>

Class 20: Wednesday March 10

SUMMARY Contour Integration

CURRENT READING Saff & Snider, §4.2

HOMEWORK Saff & Snider, Section 4.2 # 3,7,8,12,14 **Extra Credit 16, 17**

Exercise

First, let's recall how to integrate complex functions of a **real** variable. Compute the following:

$$(a) \int_1^2 \frac{-i}{t^2} + (t + 2i)^3 dt$$

$$(b) \int_0^\infty e^{-z^2 t} dt$$

Contour Integration

Integration of a complex function of a **complex** variable is performed on a set of connected points from, say, z_1 to z_2 . It is a **contour integral**. Given a contour C defined as $z(t)$ for $a \leq t \leq b$ where $z_1 = z(a)$ and $z_2 = z(b)$, an integral of a complex function of a complex variable $f(z)$ is written as

$$\int_C f(z) dz \quad \text{or} \quad \int_{z_1}^{z_2} f(z) dz$$

Let $f(z)$ be piecewise continuous on $z(t)$. If C is a **contour** then $z'(t)$ is piecewise continuous on $a \leq t \leq b$ and we can redefine the integral of $f(z)$ along C as:

$$\int_C f(z) dz = \int_a^b f[z(t)]z'(t) dt$$

Examples

Compute $\int_C \operatorname{Im} z dz$ where C is a directed line segment from $z = 0$ to $z = 1 + 2i$

ALGORITHM: (steps to be taken to complete the process of contour integration)

- 1:** write down a parametrization for C , $z(t)$
- 2:** Convert the integral into an integral in (real) t variables
- 3:** Integrate!

GROUPWORK

Compute $\int_C 2\bar{z}^2 dz$ where C is a directed line segment from $z = 2$ to $z = -2$. (Sketch the contour and evaluate the integral.)

Also evaluate $\int_C 2\bar{z}^2 dz$, this time using C being a counterclockwise circular arc from $z = 2$ to $z = -2$. (Sketch the contour and then evaluate the integral.)

Also evaluate $\int_C 2\bar{z}^2 dz$, this time using C being a clockwise circular arc from $z = 2$ to $z = -2$. (Sketch the contour and then evaluate the integral.)

Question

Does the value of a contour integral depend on the path taken?

Exercise

Show that

$$\int_{C_r} (z - z_0)^n dz = \begin{cases} 2\pi i & n = -1 \\ 0 & n \neq -1 \end{cases}$$

where n is any integer and C_r is a circle of radius r around z_0 (what is the equation of such a shape?) traversed **once** in the counter-clockwise direction. How will our results change if we reverse the direction of travel along the contour (i.e. move in a clockwise direction)?