Complex Analysis

Math 214 Spring 2004 — Fowler 112 MWF 3:30pm - 4:25pm

 — http://faculty.oxy.edu/ron/math/312/04/

Class 15: **Wednesday February 25**

SUMMARY Branch Cuts and Riemann Surfaces
CURRENT READING Saff & Snider, §3.3
HOMEWORK Saff & Snider, Section 3.3 # 1, 2, 3, 4, 5

Branching: Producing Single-valued functions from Multiple-valued ones

A single-valued function $F(z)$ is said to be a *branch* of a multiple-valued function $f(z)$ in a domain D if $F(z)$ is single-valued and analytic in D and has the property that for each $z \in D$, the value $F(z)$ is one of the values of $f(z)$

Branch cuts do not have to be along the x-axis, they can be any line which when removed from the domain of definition of the multi-valued function, produce a single valued function.

Other branches of $\log z$

One can define other analytic branches of $\log z$ by choosing different branch cuts.
The usual way to do this is to make the branch cut along $\theta = \alpha$ starting at the origin, so that

$$\log z = \ln |z| + i\theta, \qquad \text{where } \alpha < \theta \leq \alpha + 2\pi$$

These branches of $\log z$ can be denoted $\mathcal{L}_\alpha$, or $\log_\alpha$ where $\theta = \alpha$ is where the branch cut is.

A **branch point** of a function f is a point which is common to all branch cuts of f. So, 0 is a branch point of $\log z$

EXAMPLE

Determine the domain of analyticity for the function $f(z) = \text{ Log }(3z - i)$ and compute $f'(z)$
What is $f(i)$? What about $f'(i)$?

Exercise

Saff & Snider, page 124, #13

Find a branch of $\log(2z - 1)$ that is analytic at all points in the plane except for those on the following rays:

(a) $\{z = z + iy : x \leq 1/2, y = 0\}$

(b) $\{z = z + iy : x \geq 1/2, y = 0\}$

(c) $\{z = z + iy : x = 1/2, y \geq 0\}$

Riemann Surfaces

Let's look at the Riemann Surface for the function $f(z) = z^{1/2}$.
Do we understand why this function needs a branch cut?

Do we understand how the Riemann surface then acts as continuous domain for $f(z) = z^{1/2}$

Now let's turn to the Riemann surface for $\log(z)$. What are the main difference between the Riemann surface for the two functions? What do they have in common?