Complex Analysis

Math 214 Spring 2004 — Fowler 112 MWF 3:30pm - 4:25pm

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Class 13: **Friday February 20**

SUMMARY Complex Trigonometric Functions and Complex Hyperpolic Trigonometric Functions
CURRENT READING Saff & Snider, §3.2
HOMEWORK Saff & Snider, Section 3.2 # 13, 14, 15, 17, 18, **Extra Credit: #20, 23**

Complex trigonometric functions

Once we have a handle on $\exp z$ we can use it to define other functions, most notably $\sin z$ and $\cos z$

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \qquad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

EXAMPLE

Show that $\frac{d}{dz}\cos z = -\sin z$ by using the definition of $\cos z$

There a whole bunch of typical trigonometric identities which are valid for complex trig functions. Most of these can be proved using the definitions involving exponentials. For example, $\tan z$, $\sec z$ are analytic everywhere except at the zeroes of $\cos z$.

Exercise

Find the zeroes of $\cos z$ and $\sin z$

The usual rules of derivatives of the trig functions remain valid for their complex counterparts.

Complex Trigonometric Identities

$$\begin{aligned} \sin(z+2\pi) &= \sin z, & \cos(z+2\pi) &= \cos z \\ \sin(-z) &= -\sin z, & \cos(-z) &= \cos z \\ \sin^2 z + \cos^2 z &= 1, & \tan^2 z + 1 &= \sec^2 z \\ \sin 2z &= 2\sin z \cos z, & \cos 2z &= \cos^2 - \sin^2 z \\ \sec z &= \frac{1}{\cos z}, & \tan z &= \frac{\sin z}{\cos z} \\ \frac{d}{dz}\tan z = \sec^2 z,\ \frac{d}{dz}\sec z &= \sec z \tan z & \frac{d}{dz}\sin z = \cos z,\ \frac{d}{dz}\sin z &= \cos z \end{aligned}$$

Similarly the hyperbolic trigonometric functions can be defined using the complex exponential and the newly-defined complex trig functions

$$\sinh z = \frac{e^z - e^{-z}}{2}, \qquad \cosh z = \frac{e^z + e^{-z}}{2}$$

Complex Hyperbolic Trigonometric Identities

$$\begin{aligned} \sinh z &= -i \sin iz, & \cosh z &= \cos(iz) \\ \frac{d}{dz}\sinh z &= \cosh z, & \frac{d}{dz}\cosh z &= \sinh z \end{aligned}$$

GroupWork

Show that the mapping $w = \sin(z)$
(a) maps the y-axis one-to-one and onto the v-axis
(b) maps the ray $\{z : \text{ Arg } z = \pi/2\}$ one-to-one and onto the ray $\{w : \text{ Re } (w) > 1, \text{ Im } w = 0\}$

Exercise

1. Show that $\sin(\overline{z}) = \overline{\sin(z)}$

2. For what values of z does $\cos(z) = 2$?