Complex Analysis

Math 214 Spring 2004 — Fowler 112 MWF 3:30pm - 4:25pm
 — http://faculty.oxy.edu/ron/math/312/04/

Class 11: **Friday February 13**

SUMMARY Application of Complex Variables: The Mandelbrot and Julia Sets
CURRENT READING Saff & Snider, §2.6 and §2.7
HOMEWORK Saff & Snider, Section 2.7 # 5, 10

Functional Iteration

Suppose we have a function $f(x)$ and we input the value $f(x)$ into the function, to produce $f(f(x))$, and input this number into the function again, to produce $f(f(f(x)))$, and so on. The process is called **functional iteration** (also known as Picard Iteration).
What's interesting is if the iterative process has a finite limit, especially if a fixed point will be found such that $f(p) = p$

EXAMPLE

Consider $f(x) = x^2$. Execute functional iteration with $x_0 = -.5$, $x_0 = 1$ and $x_0 = 1.5$. What happens?

What are the fixed point(s) of $f(x) = x^2$? Do these fixed points different in character?

Functional Iteration on Complex Numbers

THEOREM. If (1) $f(z)$ is analytic in a neighborhood of $z = z_{fixed}$
(2) $f(z_L) = z_{fixed}$ and
(3) $|f'(z_{fixed}| < 1$ then there exists a disk around z_{fixed} with the property that all orbits launched from inside the disk remain confined to the disk and the orbits converge to z_{fixed} in the limit.

PROOF

DEFINITION

The **filled Julia Set** for a polynomial function $f(z)$ is defined to be the set of points that launch bounded orbits through functional iteration of f; the **Julia set** is the boundar of the filled Julia set.

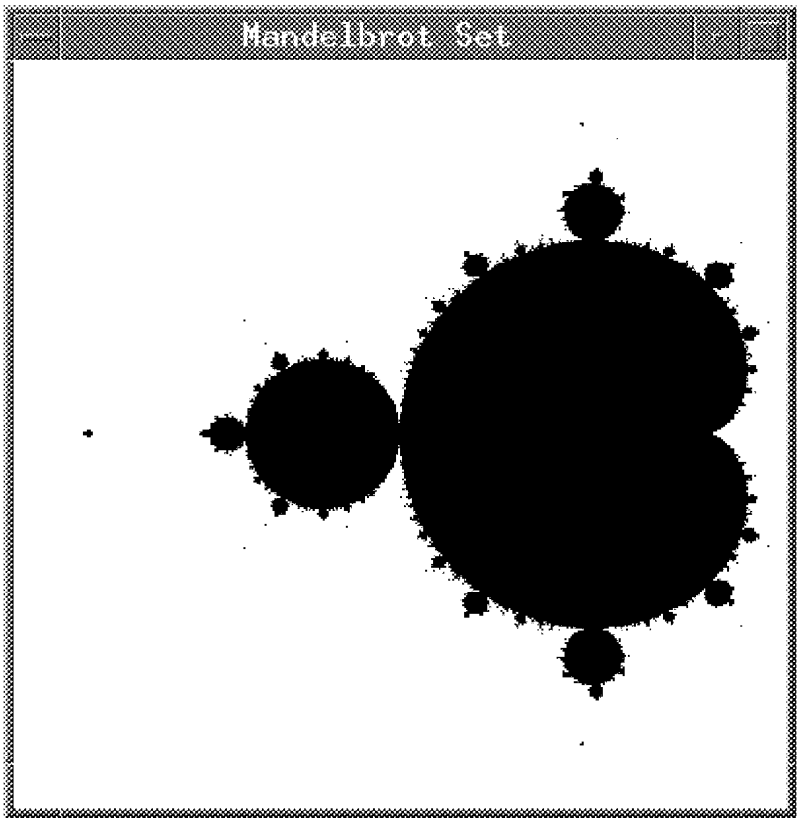
EXAMPLE 2

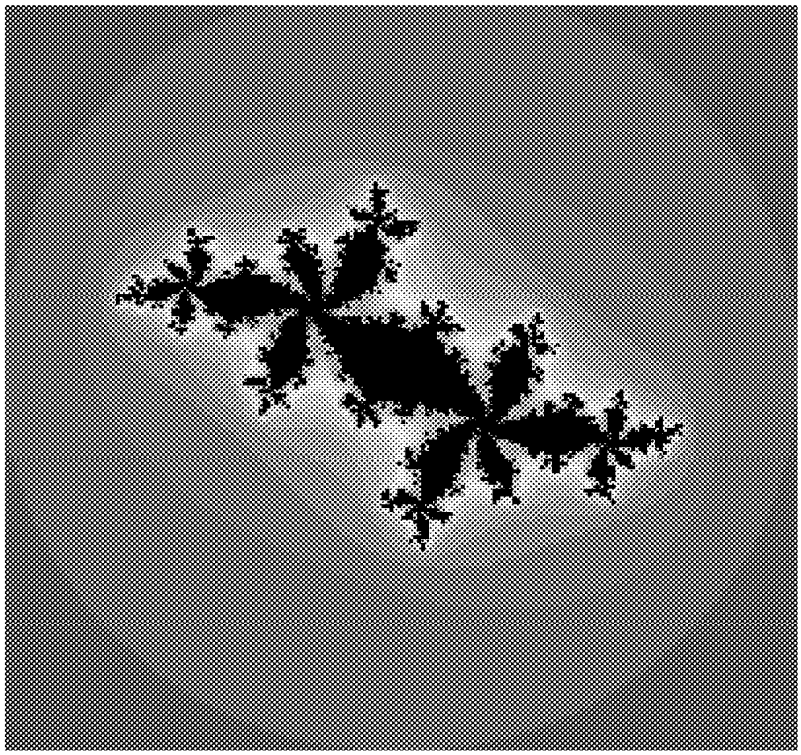
Q: What is the Julia set for $f(z) = z^2$?

A: $J = \{z : |z| = 1\}$

The Mandelbrot Set

If we consider the function $f(z) = z^2 - c$ where c is a complex number we find that the values of c which produce Julia sets which are connected and the values of c which produces Julia sets which are disconnected, themselves form a very interesting set of points in the complex plane. THIS set is called the **Mandelbrot set**





Julia Set

Let's look on the web of examples of Julia sets of other complex polynomials, especially the **Mandelbrot set**, the most famous Julia set of all time!