Bonus Quiz   Complex Analysis

Name: _____

Date: _____   **Friday February 6**
Time Begun: _____   Ron Buckmire
Time Ended: _____

Topic : Mappings and Points Sets in the Extended Argand Plane

The point of this quiz is to provide another example of using functions as mappings and to give you more practice becoming familar with describing and sketching point sets of complex numbers. In addition it also tests taking limits of and evaluating functions of a complex variable in the extended z-plane.

Instructions:

1. Once you open the quiz, you have as much time as you need to complete it, but record your start time and end time at the top of this sheet.

2. You may use the book or any of your class notes. You must work alone.

3. If you use your own paper, please staple it to the quiz before coming to class. If you don't have a stapler, buy one.

4. After completing the quiz, sign the pledge below stating on your honor that you have adhered to these rules.

5. Your solutions must have enough details such that an impartial observer can read your work and determine HOW you came up with your solution.

6. Relax and enjoy...

7. **This quiz is due on Monday, February 9**, in class. NO LATE QUIZZES WILL BE ACCEPTED.

Pledge: I, _____, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

1. The *Joukowsky mapping* is a very famous mapping used in theoretical aerodynamics to represent cross-sections of airfoils. It is denoted by $J(z)$ and defined as

$$w = J(z) = \frac{1}{2}\left(z + \frac{1}{z}\right)$$

(a) (*6 points*) Sketch the image of the circle $|z| = \rho$ under the Joukowsky mapping. (Describe the image in the w-plane using complex inequalities, and in english). One approach to this problem would be to individually map several connected points on the circle in the z-plane to the w-plane, and see how they are connected there.

(b) (*1 point*) Where does the unit circle $|z| = 1$ get mapped to?

(c) (*1 point*) Show that $J\left(\dfrac{1}{z}\right) = J(z)$, for all z.

(d) (*2 points*) Compute $\lim\limits_{z \to 0} J(z)$ and $\lim\limits_{z \to \infty} J(z)$