

Complex Analysis

Math 214 Spring 2004
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Fowler 112 MWF 3:30pm - 4:25pm
http://faculty.oxy.edu/ron/math/312/04/

Homework Set 9

30 points + 10 journal points = **40 points**

ASSIGNED: Fri Mar 26 2004

DUE: Fri April 2 2004

- (3 points) **Saff & Snider, page 202, # 13.** Evaluate $\oint 1/(z^2 + 1) dz$ along the three closed contours Γ_1, Γ_2 and Γ_3 given on page 202 in Figure 4.47.
- (4 points) **Saff & Snider, page 202, # 17.** Evaluate $\oint_{\Gamma} \frac{2z^2 - z + 1}{(z - 1)^2(z + 1)} dz$ along the fancy contour given on page 203 in Figure 4.49.
- (5 points) **Saff & Snider, page 201, # 10.** Determine the domain of analyticity for each of the given functions f (write the domain using complex set notation). Explain why $\oint_{|z|=2} f(z) dz = 0$ for each of the following functions
 - $f(z) = \frac{z}{z^2 + 25}$
 - $f(z) = e^{-z}(2z + 1)$
 - $f(z) = \text{Log}(z + 3)$
 - $f(z) = \sec\left(\frac{z}{2}\right)$
 - $f(z) = \frac{\cos(z)}{z^2 - 6z + 10}$
- (5 points) **Saff & Snider, page 213, # 5.** Let C be the ellipse $x^2/4 + y^2/9 = 1$ traversed once in the positive direction, and define

$$G(z) := \oint_C \frac{\xi^2 - \xi + 2}{\xi - z} d\xi \quad z \text{ inside } C$$

Compute $G(1)$, $G'(i)$ and $G''(-1)$

- (4 points) **Saff & Snider, page 213, # 7.** Compute $\oint_C \frac{\cos z}{z^2(z - 3)} dz$ along the contour indicated in Figure 4.55 on page 213.
- (5 points) Consider the contour $C : |z - i| = 2$ traversed counter-clockwise and show that $\oint_C \frac{1}{z^2 + 4} dz = \frac{\pi}{2}$ while $\oint_C \frac{1}{(z^2 + 4)^2} dz = \frac{\pi}{16}$
- (4 points) Let C be the unit circle $z = e^{i\theta} (-\pi \leq \theta \leq \pi)$. First show that, for any **real** constant a , $\oint_C \frac{e^{az}}{z} dz = 2\pi i$. Then re-write the integral in terms of θ to obtain the integration formula: $\int_0^\pi e^{a \cos \theta} \cos(a \sin \theta) d\theta = \pi$

JOURNAL ENTRY

(10 points) Use a separate sheet of paper to discuss the Cauchy integration formulas and Cauchy integration theorems in several paragraphs. Do you understand how the formulas work and when you can use them? What are the conditions for which the Cauchy-Goursat Theorem does not apply? You can use specific examples drawn from this homework set. Provide your overall feedback about the homework set. How long did it take you to complete? Which questions were difficult and why?

NOTES

Homework sets are due **one week** from when they are given out. You are strongly encouraged to work collaboratively on the homework and to visit me during office hours to ask questions. Each person must hand in individually-written work and indicate with whom they collaborated on the answers. On your first solution page, Indicate the names of the students you worked with.