Real Analysis

Math 310 — MWF 10:30-11:25am
Fall 1996 — 127 Fowler

Ron Buckmire ~ 402 Fowler ~ 259-2536 ~ `ron@oxy.edu` ~ Mon 1:30-3:30pm Wed & Thu 1:30-2:30pm

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#28 Monday (11/25/96)

SUMMARY Power Series

READING §23

To find a power series of $f(x)$ in *Derive*, use **C**alculus, **T**aylor. Then input the function, the variable, and the order of the series (highest power in the polynomial). As output, you will get something like: TAYLOR(f(x), x, 0, n) (Note: You can just **A**uthor the Taylor command yourself rather than going through the menu.) The 0 represents the point about which you took the power series. Let's deal with only 0 right now. You can do it about any point you wish. To actually see the Taylor series written out, you must use **E**xpand (or **S**implify, or appro**X**).

Examples. Find the power series about $x = 0$ for the following functions:

(a) $e^x =$

(b) $\sin(x) =$

(c) $\dfrac{1}{1-x} =$

(d) $\ln(1-x) =$

Let's now look at the plots of some of these functions versus the first few terms of the their Taylor series.

Plot $\sin(x)$, TAYLOR(sin(x), x, 0, 3), TAYLOR(sin(x), x, 0, 5), TAYLOR(sin(x), x, 0, 9) on the same plot. Make some observations.

Plot $\ln(1-x)$, TAYLOR(ln(1-x), x, 0, 3), TAYLOR(ln(1-x), x, 0, 5), TAYLOR(ln(1-x), x, 0, 9) on the same plot. Make some observations and contrast with those for the sine function.

Sometimes we have infinite power series, but really don't know what functions they converge to. Look at the examples in # 23.1 on page 132. Use the **C**alculus, **S**um commands to define the partial sums of some of these series. Then plot a few of the partial sums for the same series on the same plot. What can you say about the "intervals of convergence"?

Power Series

Given a sequence $(a_n)_0^\infty$, the series $\sum_{n=0}^\infty a_n x^n$ is called a POWER SERIES.

As we have seen, the power series is a function of x. It converges for $x = 0$. By convention, we take $0^0 = 1$.
Given it any sequence (a_n), $\sum a_n x^n$ either

- CONVERGES for all $x \in \mathbb{R}$
- CONVERGES for only $x = 0$
- CONVERGES for all x in some bounded interval

Radius Of Convergence

23.1 For the power series $\sum a_n x^n$, let $\beta = \limsup |a_n|^{1/n}$ and $R = 1/\beta$
(If $\beta = 0 \to R = +\infty$ but if $\beta = +\infty \to R = 0$) The power series either

- CONVERGES for $|x| < R$
- DIVERGES for $|x| > R$

For $|x| = R$ (i.e. $x = -R$ and $x = R$) convergence of the power series must be tested separately.

R is called the **radius of convergence**.

PROOF
CASE 1: $0 < R < \infty$

CASE 2: $R = +\infty$

CASE 3: $R = 0$

Remember: even though $\lim sup|a_n|^{1/n}$ can be difficult to compute, if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$ exists and equals L, then

$$L = \lim |a_n|^{1/n} = \limsup |a_n|^{1/n}$$

Homework Assignment (Due Monday December 2)

§23, page 132: **23.1, 23.4**