Real Analysis

Math 310 | MWF 10:30-11:25am
Fall 1996 | 127 Fowler

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#26 Monday (11/20/96)

SUMMARY Uniform Continuity

READING §19 (page 102-110)

Warm-Up
Write down the definition of UNIFORM CONTINUITY below...

Theorem
19.2 If f is continuous on a closed interval [a,b], then f is uniformly continuous on $[a, b]$.

USEFULNESS OF UNIFORM CONTINUITY

Theorem
19.4 If f is uniformly continuous on a set S and (s_n) is a Cauchy sequence in S, then $(f(s_n))$ is a Cauchy sequence.

This theorem can be used to show (again) that $f(x) = 1/x^2$ is not uniformly continuous on $(0, 1)$

Definition

A function $\tilde{f}$ is said to be an **extension of** f if $\mathrm{dom}(f) \subseteq \mathrm{dom}(\tilde{f})$ and $f(x) = \tilde{f}(x)$ for all $x \in \mathrm{dom}(f)$

If $f(x) = \sin(1/x)$ for $x \in (0, 1/\pi]$ Observe this extension of f written below:

$$\tilde{f}(x) = \begin{cases} x\sin(1/x) & \text{for} \quad 0 < x \leq 1/\pi \\ 0 & \text{for} \quad x = 0 \end{cases}$$

What is $\mathrm{dom}(f)$?

What is $\mathrm{dom}(\tilde{f})$?

Is $\tilde{f}$ continuous at $x = 0$?

Theorem

19.5 A real-valued function on (a,b) is uniformly continuous on (a,b) if and only if it can be extended to a continuous function $\tilde{f}$ on [a,b]

Theorem

19.6 Let f is a continuous function on an interval I (I may be bounded or unbounded). Let I^0 is the interval unbounded by removing any endpoints of I which happen to be in I. If f is **differentiable** on I^0 and if f' is **bounded on** I^0, then f is **uniformly continuous** on I.

PROOF: Let's read thru page 108 of the text.

Homework Assignment (Due Friday November 22)

§19, page 109: **19.1, 19.2, 19.4(a), 19.5(d),(e),(f)**